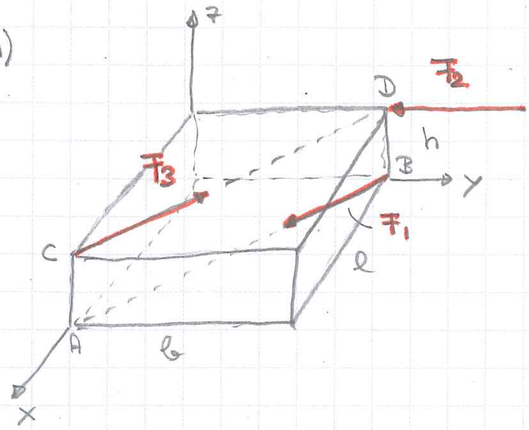
Musterlösung Zusatzblatt:

A1)



Kraftvektoren:

$$\underline{F}_1 = F \cdot \underline{e}_{AB} \qquad \underline{e}_{AB} = \frac{\underline{r}_{AB}}{|\underline{r}_{AB}|} = \frac{\begin{pmatrix}6\\-6\\0\end{pmatrix}m}{8.49m} = \begin{pmatrix}0.707\\-0.707\\0\end{pmatrix}, \quad \underline{r}_{AB} = \begin{pmatrix}l\\0\\0\end{pmatrix} - \begin{pmatrix}0\\b\\0\end{pmatrix} = \begin{pmatrix}l\\-b\\0\end{pmatrix} = \begin{pmatrix}6\\-6\\0\end{pmatrix}m$$

$$|\underline{r}_{AB}| = \sqrt{6^2+6^2} = \sqrt{72} = 8.49m$$

$$\underline{\underline{F_1 = \begin{pmatrix}7.07\\-7.07\\0\end{pmatrix} N}}$$

$$\underline{\underline{F_2 = F \cdot (-\underline{e}_y) = \begin{pmatrix}0\\-10\\0\end{pmatrix} N}}$$

$$\underline{F}_3 = F\, \underline{e}_{BA} \qquad \underline{e}_{BA} = -\underline{e}_{AB} = \begin{pmatrix}-0.707\\0.707\\0\end{pmatrix}$$

$$\underline{\underline{F_3 = \begin{pmatrix}-7.07\\7.07\\0\end{pmatrix} N}}$$

$$\underline{\underline{F_{res}}} = \underline{F}_1 + \underline{F}_2 + \underline{F}_3 = \begin{pmatrix}0\\-10\\0\end{pmatrix} N = \underline{\underline{-10\,\underline{e}_y N}} \qquad \text{res. Kraft}$$

Moment im Ursprung:

$\underline{r}_B = b\,\underline{e}_y$
$\underline{r}_C = l\,\underline{e}_x + h\,\underline{e}_z$
$\underline{r}_D = b\,\underline{e}_y + h\,\underline{e}_z$

$$\underline{M}_0 = \underline{r}_B \times \underline{F}_1 + \underline{r}_C \times \underline{F}_3 + \underline{r}_D \times \underline{F}_2$$

$$= \begin{vmatrix} e_x & e_y & e_z \\ 0 & 6 & 0 \\ 7.07 & -7.07 & 0 \end{vmatrix} + \begin{vmatrix} e_x & e_y & e_z \\ 6 & 0 & 2 \\ -7.07 & 7.07 & 0 \end{vmatrix} + \begin{vmatrix} e_x & e_y & e_z \\ 0 & 6 & 2 \\ 0 & -10 & 0 \end{vmatrix} = \begin{pmatrix}-14.14+20\\-14.14\\-42.42+42.42\end{pmatrix} = \underline{\underline{\begin{pmatrix}5.86\\-14.14\\0\end{pmatrix} Nm}}$$

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Kraftschraube:

Moment, welches in selbe Richtung wirkt wie $\underline{F}_{Res}$!

$\underline{M}_{res} \parallel \underline{F}_{res}$!

$\underline{M}_{\perp} + \underline{M}_{res} \stackrel{!}{=} \underline{M}_0$ Forderungen

Verschiebungsvektor zu Ursprung:

$\underline{M}_{\perp} = \underline{a} \times \underline{F}_{res} = \underline{M}_0 - \underline{M}_{res}$ von oben eingesetzt

nicht einfach auflösbar, daher:

$$\underline{F}_{res} \times (\underline{a} \times \underline{F}_{res}) = \underline{F}_{res} \times (\underline{M}_0 - \underline{M}_{res}) = \underline{F}_{res} \times \underline{M}_0 - \underbrace{\underline{F}_{res} \times \underline{M}_{res}}_{=0 \text{ weil } \underline{M}_{res} \parallel \underline{F}_{res}!}$$

und
$$\underline{F}_{res} \times (\underline{a} \times \underline{F}_{res}) = (\underline{F}_{res} \circ \underline{F}_{res})\, \underline{a} - \underbrace{(\underline{F}_{res} \circ \underline{a})}_{=0 \text{ weil } \underline{F}_{res} \perp \underline{a}} \underline{F}_{res}$$

also
$$\boxed{F_{res}^2\, \underline{a} = \underline{F}_{res} \times \underline{M}_0}$$

$$\underline{a} = \frac{\underline{F}_{res} \times \underline{M}_0}{F_{res}^2} = \frac{\begin{vmatrix} \underline{e}_x & \underline{e}_y & \underline{e}_z \\ 0 & -10 & 0 \\ 5.86 & -14.14 & 0 \end{vmatrix} \mathrm{N^2 m}}{100\,\mathrm{N^2}} = \frac{\begin{pmatrix} 0 \\ 0 \\ 58.6 \end{pmatrix} \mathrm{m}}{100} = \underline{\underline{0.586\, \underline{e}_z\, \mathrm{m}}}$$

Moment der Kraftschraube ist Projektion auf $\underline{F}_{res}$-Richtung:

$$\underline{e}_{res} = \frac{\underline{F}_{res}}{|\underline{F}_{res}|} = \frac{-10\, \underline{e}_y\, \mathrm{N}}{10\, \mathrm{N}} = -\underline{e}_y$$

also: $\underline{M}_{res} = \underline{M}_0 \circ -\underline{e}_y = \underline{\underline{+14.14\, \underline{e}_y\, \mathrm{Nm}}}$

A2) Res. Kraft:

$\underline{F}_{Res} = 2\underline{F}_R + 2\underline{F}_D + 2\underline{F}_E + 2\underline{F}_L$

$= 2\left[35N + 45N + 32N + 23N\right]\underline{e}_z$

$\underline{F}_{Res} = 270\,\underline{e}_z\,N$

$$\underline{F}_{Res} = \begin{pmatrix} 0 \\ 0 \\ 270 \end{pmatrix} N$$

Res. Moment:

Symmetrie beachten: nur y-Abstände relevant

$\underline{M}_O = 2\left[\underline{r}_R \times \underline{F}_R + \underline{r}_E \times \underline{F}_E + \underline{r}_L \times \underline{F}_L\right]$

$= 2\left[-0.075 \cdot 35 + 0.015 \cdot 32 + 0.045 \cdot 23\right]\underline{e}_x\,Nm$

wobei $\underline{r}_R = -0.075m\,\underline{e}_y$

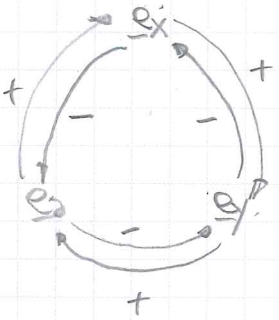
$\underline{r}_E = 0.015m\,\underline{e}_y$

$\underline{r}_L = 0.045m\,\underline{e}_y$

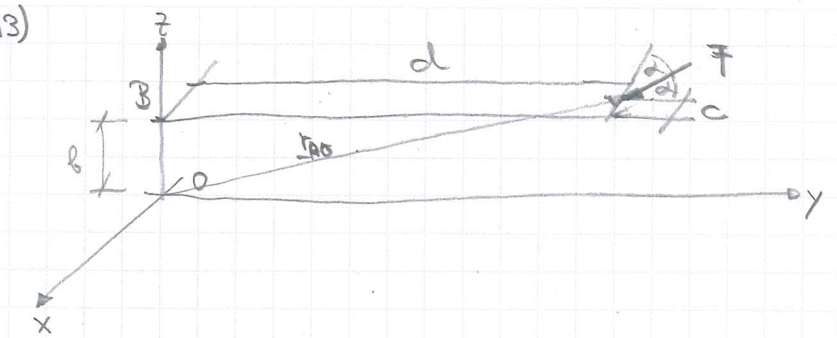
sowie $\underline{e}_y \times \underline{e}_z = \underline{e}_x$ f. alle KP

$\underline{M}_O = -2.22\,\underline{e}_x\,Nm$

$$\underline{M}_O = \begin{pmatrix} -2.22 \\ 0 \\ 0 \end{pmatrix} Nm$$



A3)



Kraftvektor $\underline{F}$:

$$\underline{F} = F \cdot (\sin\alpha\, \underline{e}_x - \cos\alpha\, \underline{e}_y) = \left(\frac{30}{\sqrt{2}} \underline{e}_x - \frac{30}{\sqrt{2}} \underline{e}_y\right) N$$

Abstandsvektor:

$$\underline{r}_{AO} = -c\, \underline{e}_x + d\, \underline{e}_y + b\, \underline{e}_z = (-10 \underline{e}_x + 200 \underline{e}_y + 50 \underline{e}_z)\, mm$$

Momentenvektor um O:

$$\underline{M}_O = \underline{r}_{AO} \times \underline{F} = (-10 \underline{e}_x + 200 \underline{e}_y + 50 \underline{e}_z) \times \left(\frac{30}{\sqrt{2}} \underline{e}_x - \frac{30}{\sqrt{2}} \underline{e}_y\right)$$

$\underline{e}_z$, $-\underline{e}_z$, $\varepsilon_{zxy} \to \underline{e}_y$, $\varepsilon_{zyx} \to -\underline{e}_x$

$$= \left(+\frac{300}{\sqrt{2}} \underline{e}_z - \frac{6000}{\sqrt{2}} \underline{e}_z + \frac{1500}{\sqrt{2}} \underline{e}_y + \frac{1500}{\sqrt{2}} \underline{e}_x\right) Nmm$$

$$\underline{M}_O = \left(+\frac{1.5}{\sqrt{2}} \underline{e}_x + \frac{1.5}{\sqrt{2}} \underline{e}_y - \frac{5.7}{\sqrt{2}} \underline{e}_z\right) Nm = \underline{(+1.06 \underline{e}_x + 1.06 \underline{e}_y - 4.03 \underline{e}_z)\, Nm}$$

z-Komponente:

$$M_z = \underline{M}_O \cdot \underline{e}_z = \underline{-4.03\ Nm} \quad , \quad |\underline{M}_O| = \sqrt{1.06^2 + 1.06^2 + 4.03^2} = \underline{4.3\ Nm}$$

Koordinatenwinkel

$$\alpha = \arccos\left(\frac{1.06}{4.3}\right) = \underline{75.7^\circ} = \beta$$

$$\gamma = \arccos\left(\frac{-4.03}{4.3}\right) = \underline{160^\circ}$$