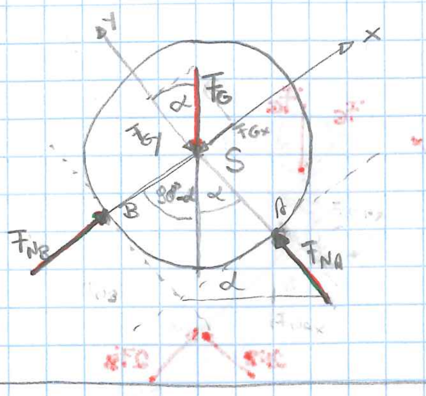
Bsp 8:

r = 350 mm
$\alpha = 30^\circ$
m = m



$F_G = m \cdot g$ Gewichtskraft (Gravitation)

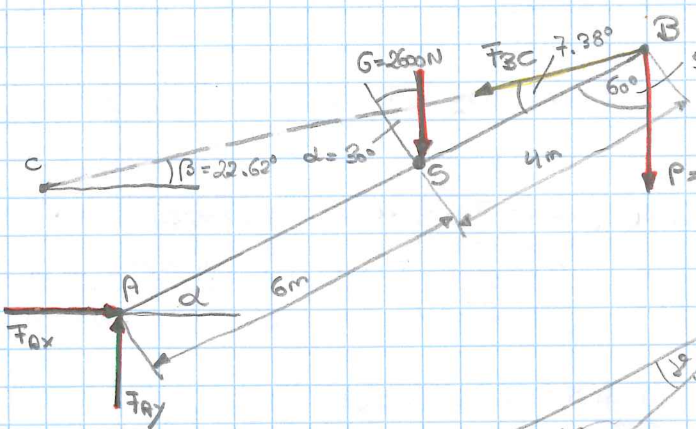
$F_{Gx} = m \cdot g \cdot \sin\alpha$
$F_{Gy} = m \cdot g \cdot \cos\alpha$

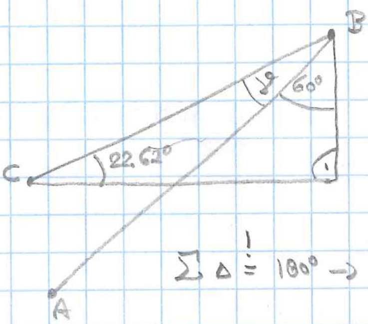
$F_{NA} = F_{Gy}$ $\quad \sum_i F_{y_i} = 0$
$F_{NB} = F_{Gx}$ $\quad \sum_i F_{x_i} = 0$

} Normalkräfte (Reaktionskräfte) der Dauben auf die Papierrolle.

Bsp 9:

G = 2600 N
P = 5000 N
$l_1 = 6\,m$
$l_2 = 4\,m$
$\alpha = 30^\circ$
$\tan\beta = \frac{5}{12}$
$\beta = 22{,}62^\circ$





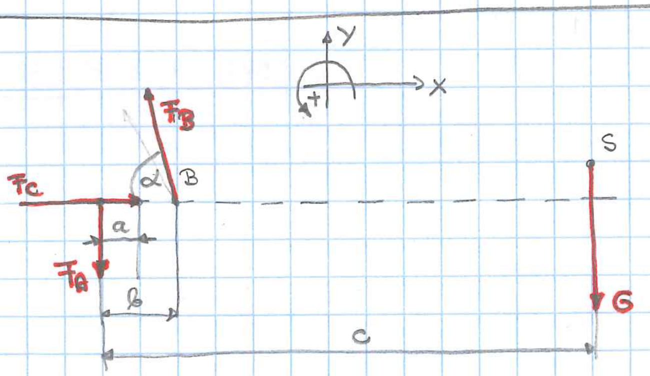
G ... Gewichtskraft d. Kranauslegers
F_{Ax}, F_{Ay} ... Lagerreaktion des Lagers A auf Ausleger
F_{BC} ... Zugkraft des Seils auf Ausleger in B
P ... Gewichtskraft der Last auf Ausleger in B

$\sum \Delta \overset{!}{=} 180^\circ \rightarrow 90^\circ$ frei

$\gamma = 90^\circ - 60^\circ - 22.62^\circ$
$\underline{\gamma = 7.38^\circ}$

Bsp 10:

G = 20 N
a = 20 mm
b = 50 mm
c = 350 mm
$\alpha = 75^\circ$



$\sum_i M_{B_i} = 0: \quad F_A \cdot b - G \cdot (c - b) = 0$ (1)

$\sum_i F_{y_i} = 0: \quad -F_A + F_B \cdot \sin\alpha - G = 0$ (2)

$\sum_i F_{x_i} = 0: \quad F_C - F_B \cdot \cos\alpha = 0$ (3)

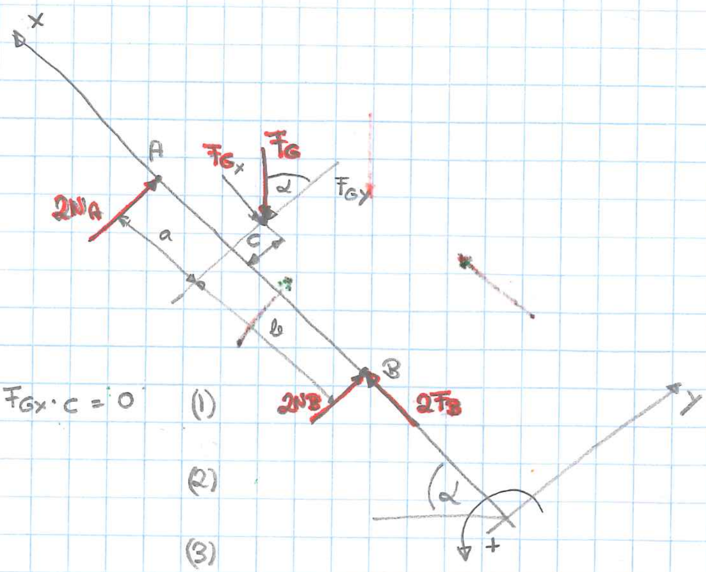
aus (1): $\underline{F_A} = \frac{G \cdot (c - b)}{b} = \frac{20N \cdot (350mm - 50mm)}{50mm} = \underline{120\,N}$

aus (2): $\underline{F_B} = \frac{G + F_A}{\sin\alpha} = \frac{20N + 120N}{\sin(75^\circ)} = \underline{144.9\,N}$

aus (3): $\underline{F_C} = F_B \cdot \cos\alpha = 144.9N \cdot \cos(75^\circ) = \underline{37.51\,N}$

Bsp. 11:

$m = 1500\,kg$
$g = 9.81\,m s^{-2}$
$c_A = 58\,kN m^{-1}$
$c_B = 65\,kN m^{-1}$
$a = 0.8\,m$
$b = 1.2\,m$
$c = 0.4\,m$
$\alpha = 30°$



$\sum_i M_{B_i} = 0: \quad -2N_A \cdot (a+b) + F_{Gy} \cdot b - F_{Gx} \cdot c = 0 \quad (1)$

$\sum_i F_{x_i} = 0: \quad 2F_B - F_{Gx} = 0 \quad (2)$

$\sum_i F_{y_i} = 0: \quad 2N_A + 2N_B - F_{Gy} = 0 \quad (3)$

aus (1): $2N_A \cdot (a+b) = F_G \cdot \cos\alpha \cdot b - F_G \cdot \sin\alpha \cdot c$, $F_G = m \cdot g = 1500\,kg \cdot 9.81\,ms^{-2} = \underline{14715\,N}$ $(kg\,m\,s^{-2})$

$$N_A = \frac{F_G \cdot \cos\alpha \cdot b - F_G \cdot \sin\alpha \cdot c}{2(a+b)} = \frac{14715\,N(\cos(30°) \cdot 1.2\,m - \sin(30°) \cdot 0.4\,m)}{2\,(0.8\,m + 1.2\,m)}$$

$$\underline{N_A} = 3087.32\,N = \underline{3.087\,kN}$$

aus (2): $\underline{F_B} = \frac{F_G \cdot \sin\alpha}{2} = \frac{14715\,N \cdot \sin(30°)}{2} = 3678.75\,N = \underline{3.68\,kN}$

Reibkraft auf jedes Hinterrad

aus (3): $N_B = \frac{F_G \cdot \cos\alpha - 2N_A}{2} = \frac{14715\,N \cdot \cos(30°) - 2 \cdot 3087.32\,N}{2}$

$$\underline{N_B} = 3284.46\,N = \underline{3.28\,kN}$$

Federkompression:

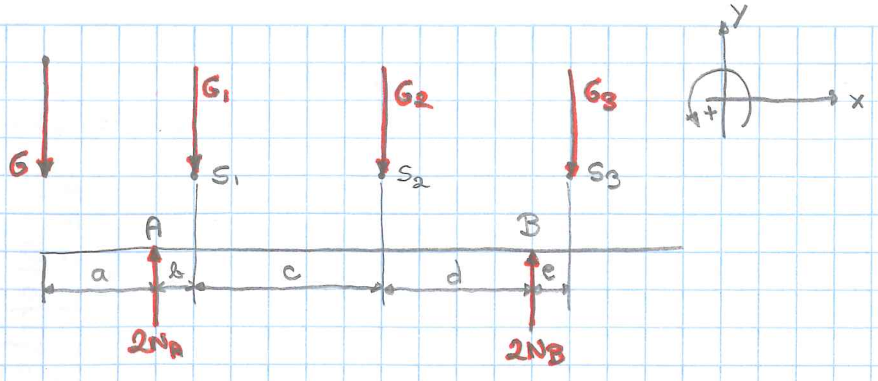
$$X = \frac{F_{sp}}{c}$$

also: $\underline{X_A} = \frac{N_A}{c_A} = \frac{3.087\,kN}{58\,kN m^{-1}} = 0.0532\,m = \underline{53.2\,mm}$

$\underline{X_B} = \frac{N_B}{c_B} = \frac{3.28\,kN}{65\,kN m^{-1}} = 0.0505\,m = \underline{50.5\,mm}$

Bsp. 12:

$G = 3200\,N$
$G_1 = 14000\,N$
$G_2 = 3600\,N$
$G_3 = 6000\,N$
$a = 2.5\,m$
$b = 0.75\,m$
$c = 2\,m$
$d = 1.5\,m$
$e = 0.25\,m$



$$\sum_i M_{Ai} = 0 : \quad G \cdot a - G_1 \cdot b - G_2 \cdot (b+c) - G_3 \cdot (b+c+d+e) + 2N_B (b+c+d) = 0 \qquad (1)$$

$$\sum_i F_{yi} = 0 : \quad -G + 2N_A - G_1 - G_2 + 2N_B - G_3 = 0 \qquad (2)$$

aus (1): $$N_B = \left[-G \cdot a + G_1 \cdot b + G_2 \cdot (b+c) + G_3 \cdot (b+c+d+e) \right] \frac{1}{2(b+c+d)}$$

$$N_B = \left[-G \cdot 2.5m + 14000N \cdot 0.75m + 3600N \cdot (0.75m + 2m) + 6000N \cdot (0.75m + 2m + 1.5m + 0.25m) \right] \frac{1}{2 \cdot (0.75m + 2m + 1.5m)}$$

$$\underline{N_B = 5576.47N - 0.2941 \cdot G}$$

aus (2): $$N_A = (G + G_1 + G_2 - 2N_B + G_3) \frac{1}{2}$$

$$= \frac{G}{2} + \frac{14000N}{2} + \frac{3600N}{2} - 5576.47N + 0.2941\,G + \frac{6000N}{2}$$

$$\underline{N_A = 6226.53\,N + 0.7941\,G}$$

(a) mit $G = 3200N$ erhalten wir

$$\underline{N_B = 4.632\,kN}$$

$$\underline{N_A = 8.77\,kN}$$

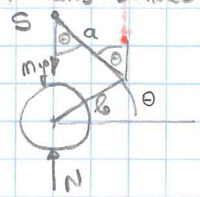
(b) Der Kran beginnt zu kippen, wenn $N_B = 0$ wird, also

$$N_B = 0 = 5576.47N - 0.2941\,G$$

$$\rightarrow \underline{G} = \frac{5576.47N}{0.2941} = \underline{18.96\,kN}$$

Bsp. 13:

Der Schwerpunkt muss mindestens senkrecht über der Radachse stehen um ein Kippen zu ermöglichen.

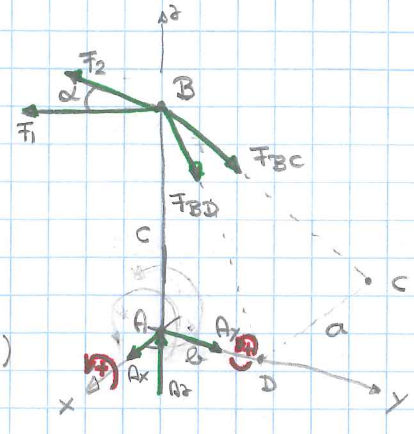


$$\tan \Theta_{max} = \frac{b}{a}$$

$$\underline{\Theta_{max} = \tan^{-1} \frac{b}{a}}$$

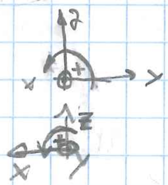
Bsp. 14: Komponentenweise gerechnet

$F_1 = 140\ kN$
$F_2 = 75\ kN$
$a = 10\ m$
$b = 5\ m$
$c = 15\ m$
$\alpha = 30°$



$B(0|0|15)$, $C(-10|5|0)$, $D(0|5|0)$

$$\underline{r}_{DB} = \begin{pmatrix}0\\5\\0\end{pmatrix} m - \begin{pmatrix}0\\0\\15\end{pmatrix} m = \begin{pmatrix}0\\5\\-15\end{pmatrix} m, \quad |\underline{r}_{DB}| = \sqrt{5^2+15^2}\ m = \sqrt{250}\ m = 5\cdot\sqrt{10}\ m$$



$$\underline{F}_{BD} = \frac{F_{BD}}{5\sqrt{10}} \begin{pmatrix}0\\5\\-15\end{pmatrix} = \underline{\frac{F_{DB}}{\sqrt{10}} \begin{pmatrix}0\\1\\-3\end{pmatrix}}$$

$$\underline{r}_{CB} = \begin{pmatrix}-10\\5\\0\end{pmatrix} m - \begin{pmatrix}0\\0\\15\end{pmatrix} m = \begin{pmatrix}-10\\5\\-15\end{pmatrix} m, \quad |\underline{r}_{CB}| = \sqrt{10^2+5^2+15^2}\ m = \sqrt{350}\ m$$

$$\underline{\underline{F}_{BC} = \frac{F_{CB}}{\sqrt{350}} \begin{pmatrix}-10\\5\\-15\end{pmatrix}}$$

$$\sum_i M_{x_i}^{(A)} = 0: \quad +F_1 \cdot \cos\alpha \cdot \cancel{c} + F_2 \cdot \cancel{c} - F_{BD_y} \cdot \cancel{c} - F_{BC_y} \cdot \cancel{c} = 0 \qquad (1)$$

$$\sum_i M_{y_i}^{(A)} = 0: \quad -F_1 \cdot \sin\alpha \cdot \cancel{c} + \underbrace{\cancel{F_{BD_x}}}_{=0} \cdot \cancel{c} + F_{BC_x} \cdot \cancel{c} = 0 \qquad (2)$$

$$\sum_i F_{x_i} = 0: \quad A_x + F_1 \cdot \sin\alpha + F_{BD_x} + F_{BC_x} = 0 \qquad (3)$$

$$\sum_i F_{y_i} = 0: \quad A_y - F_2 + F_{BD_y} + F_{BC_y} - F_1 \cdot \cos\alpha = 0 \qquad (4)$$

$$\sum_i F_{z_i} = 0: \quad A_z + F_{BD_z} + F_{BC_z} = 0 \qquad (5)$$

aus (2): $F_1 \cdot \sin\alpha + \cancel{F_{BD} \cdot \frac{0}{\sqrt{10}}} - F_{BC} \frac{10}{\sqrt{350}} = 0$

$$F_{BC} = F_1 \cdot \sin\alpha \cdot \frac{\sqrt{350}}{10} = 140\ kN \cdot \sin(30°) \cdot \frac{\sqrt{350}}{10}$$

$$\underline{F_{BC} = 130.96\ kN}$$

aus (1): $-F_1 \cdot \cos\alpha \cdot - F_2 \cdot + F_{DB} \frac{1}{\sqrt{10}} \cdot + F_{CB} \frac{5}{\sqrt{350}} \cdot = 0$

$$F_{DB} = \left(F_1 \cos\alpha + F_2 - F_{CB} \frac{5}{\sqrt{350}}\right)\sqrt{10} = \left(140\,KN \cdot \cos(30^\circ) + 75\,KN - 130.96\,KN \frac{5}{\sqrt{350}}\right)\sqrt{10}$$

$\underline{F_{DB} = 509.90\,KN}$

! BA ist ein Zweikraftstab, somit muss $A_x = A_y = 0$ gelten !

aus (3): $A_x = -F_1 \sin\alpha - \cancel{F_{DB} \frac{0}{\sqrt{10}}} + F_{CB} \frac{10}{\sqrt{350}}$

$$A_x = -140\,KN \sin 30^\circ + 130.96\,KN \frac{10}{\sqrt{350}}$$

$\underline{A_x} = 0.001\,KN \approx \underline{0\,KN}$ (Rundung!)

aus (4): $A_y = F_2 - F_{DB} \frac{1}{\sqrt{10}} - F_{CB} \frac{5}{\sqrt{350}} + F_1 \cdot \cos\alpha$

$$= 75\,KN - 509.90\,KN \frac{1}{\sqrt{10}} - 130.96\,KN \frac{5}{\sqrt{350}} + 140\,KN \cdot \cos(30^\circ)$$

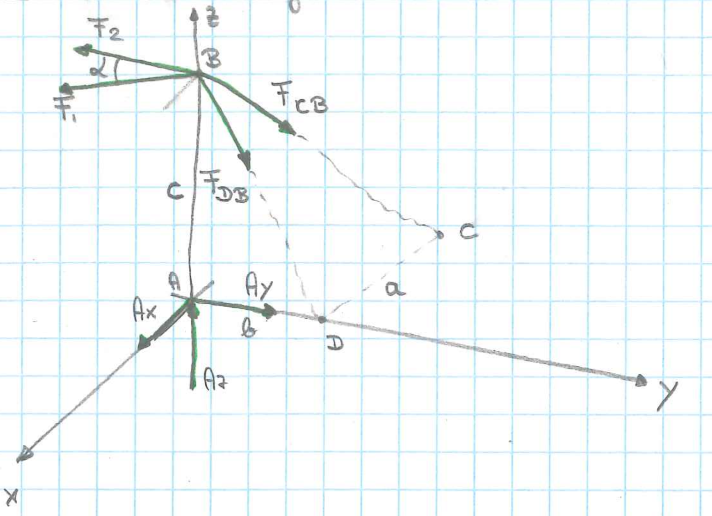
$\underline{A_y} = -0.0015\,KN \approx \underline{0\,KN}$ (Rundung!)

aus (5): $A_z = -F_{DB} \frac{-3}{\sqrt{10}} - F_{CB} \frac{-15}{\sqrt{350}}$

$$= 509.90\,KN \frac{3}{\sqrt{10}} + 130.96\,KN \frac{15}{\sqrt{350}}$$

$\underline{A_z = 586.74\,KN}$

Bsp. 14 rein vektoriell gerechnet



$$\underline{F}_{DB} = \frac{F_{DB}}{\sqrt{10}}\begin{pmatrix}0\\1\\-3\end{pmatrix}, \quad \underline{F}_{CB} = \frac{F_{CB}}{\sqrt{350}}\begin{pmatrix}-10\\5\\-15\end{pmatrix}, \quad \underline{F}_1 = F_1\begin{pmatrix}\sin\alpha\\-\cos\alpha\\0\end{pmatrix}, \quad \underline{F}_2 = F_2\begin{pmatrix}0\\-1\\0\end{pmatrix}$$

Kraftbilanz:

$$\underline{F}_{DB} + \underline{F}_{CB} + \underline{F}_1 + \underline{F}_2 + \underline{A} = \underline{0} \qquad (1) \qquad 3 \text{ Gl.}$$

Momentenbilanz um A:

$$\underline{r}_{BA} \times \underline{F}_1 + \underline{r}_{BA} \times \underline{F}_2 + \underline{r}_{BA} \times \underline{F}_{DB} + \underline{r}_{BA} \times \underline{F}_{CB} = \underline{0} \qquad (2) \qquad 2 \text{ unabh. Gl.}$$

wobei $\underline{r}_{BA} = \begin{pmatrix}0\\0\\c\end{pmatrix}$

also: $\underline{r}_{BA} \times \underline{F}_1 = \begin{vmatrix} \underline{e}_x & \underline{e}_y & \underline{e}_z \\ 0 & 0 & c \\ F_1\sin\alpha & -F_1\cos\alpha & 0 \end{vmatrix} = \begin{pmatrix}F_1\cos\alpha\cdot c\\F_1\sin\alpha\cdot c\\0\end{pmatrix}$

$$\underline{r}_{BA} \times \underline{F}_2 = \begin{vmatrix} \underline{e}_x & \underline{e}_y & \underline{e}_z \\ 0 & 0 & c \\ 0 & -F_2 & 0 \end{vmatrix} = \begin{pmatrix}F_2\cdot c\\0\\0\end{pmatrix}$$

$$\underline{r}_{BA} \times \underline{F}_{DB} = \begin{vmatrix} \underline{e}_x & \underline{e}_y & \underline{e}_z \\ 0 & 0 & c \\ 0 & 1 & -3 \end{vmatrix} \cdot \frac{F_{DB}}{\sqrt{10}} = \begin{pmatrix}-c\\0\\0\end{pmatrix}\frac{F_{DB}}{\sqrt{10}}$$

$$\underline{r}_{BA} \times \underline{F}_{CB} = \begin{vmatrix} \underline{e}_x & \underline{e}_y & \underline{e}_z \\ 0 & 0 & c \\ -10 & 5 & -15 \end{vmatrix} \cdot \frac{F_{CB}}{\sqrt{350}} = \begin{pmatrix}-5c\\-10c\\0\end{pmatrix}\frac{F_{CB}}{\sqrt{350}}$$

aus (1): $-\frac{10}{\sqrt{350}} F_{CB} + F_1 \sin\alpha + A_x = 0$ (3)

$\frac{1}{\sqrt{10}} F_{DB} + \frac{5}{\sqrt{350}} F_{CB} - F_1 \cos\alpha - F_2 + A_y = 0$ (4)

$\frac{-3}{\sqrt{10}} F_{DB} - \frac{15}{\sqrt{350}} F_{CB} + A_z = 0$ (5)

aus (2): $F_1 \cos\alpha \cdot \cancel{c} + F_2 \cdot \cancel{c} - \frac{1}{\sqrt{10}} F_{DB} \cdot \cancel{c} - \frac{5}{\sqrt{350}} F_{CB} \cdot \cancel{c} = 0$ (6)

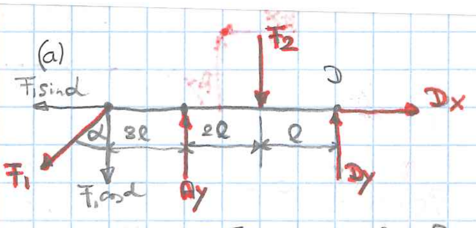
$F_1 \sin\alpha \cdot \cancel{c} - \frac{10}{\sqrt{350}} F_{CB} \cdot \cancel{c} = 0$ (7)

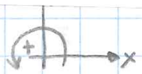
5 Unbekannte, 5 Gg. ✓

aus (7) $F_{CB} = \frac{F_1 \sin\alpha}{10} \sqrt{350}$

$\underline{\underline{F_{CB} = 130.96\ kN}}$ .... weitere Ergebnisse siehe oben

Bsp. 15: Gerberträger





$F_1 = 4\,kN$

$F_2 = 8\,kN$

$F_3 = 12\,kN$

$M = 15\,kNm$

$\ell = 2\,m$

$\alpha = 30^\circ$

$\tan\beta = \frac{4}{3} \rightarrow \beta = 53.13^\circ$

$\sum_i M_{D_i} = 0: \quad F_2 \cdot \ell + F_1 \cdot \cos\alpha \cdot 6\ell - A_y \cdot 3\ell = 0 \qquad (1)$

$\sum_i F_{y_i} = 0: \quad A_y - F_1 \cdot \cos\alpha - F_2 + D_y = 0 \qquad (2)$

$\sum_i F_{x_i} = 0: \quad D_x - F_1 \cdot \sin\alpha = 0 \qquad (3)$

aus (3) $\quad \underline{D_x} = F_1 \cdot \sin\alpha = 4\,kN \cdot \sin 30^\circ = \underline{2\,kN}$

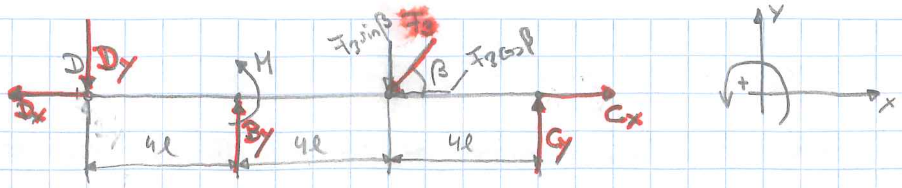
aus (1) $\quad A_y = (F_2 \cdot \cancel{\ell} + F_1 \cdot \cos\alpha\, 6\cancel{\ell}) \frac{1}{3\cancel{\ell}} = (8\,kN + 4\,kN \cdot \cos 30^\circ \cdot 6) \frac{1}{3}$

$\underline{A_y = 9.59\,kN}$

aus (2) $\quad D_y = F_1 \cos\alpha + F_2 - A_y = 4\,kN \cos(30^\circ) + 8\,kN - 9.59\,kN$

$\underline{D_y = 1.87\,kN}$

(b)



$$\sum_i M_{C_i} = 0: \quad +D_y \cdot 12l - B_y \cdot 8l + F_3 \cdot \sin\beta \cdot 4l + M = 0 \qquad (4)$$

$$\sum_i F_{y_i} = 0: \quad -D_y + B_y - F_3 \cdot \sin\beta + C_y = 0 \qquad (5)$$

$$\sum_i F_{x_i} = 0: \quad -D_x - F_3 \cdot \cos\beta + C_x = 0 \qquad (6)$$

aus (6) $\quad C_x = F_3 \cdot \cos\beta + D_x = 12\,KN \cdot \cos(53.13^\circ) + 2\,KN$

$\underline{C_x = 9.2\,KN}$

aus (4) $\quad B_y = \left(+D_y \cdot 12l + F_3 \cdot \sin\beta \cdot 4l + M\right)\dfrac{1}{8l}$

$$= \left(+1.87\,KN \cdot 12 \cdot 2m + 12\,KN \cdot \sin(53.13^\circ) \cdot 4 \cdot 2m + 15\,KNm\right)\frac{1}{8 \cdot 2m}$$

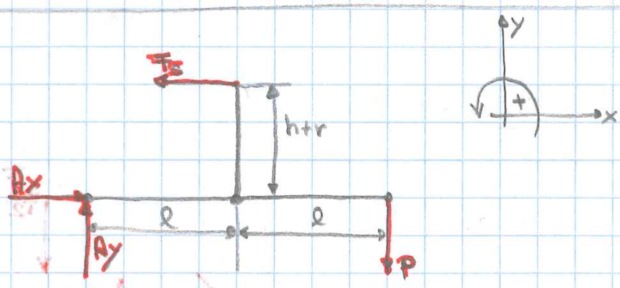
$\underline{B_y = 8.54\,KN}$

aus (5) $\quad C_y = D_y - B_y + F_3 \cdot \sin\beta = 1.87\,KN - 8.54\,K + 12\,KN \cdot \sin(53.13^\circ)$

$\underline{C_y = 2.93\,KN}$

Bsp. 15:

$F_{max} = 2\,KN$
$l = 0.75\,m$
$h = 0.5\,m$
$r = 0.1\,m$



$$\sum_i M_{A_i} = 0: \quad F_S \cdot (h+r) - P \cdot 2l = 0 \qquad (1)$$

$$\sum_i F_{x_i} = 0: \quad A_x - F_S = 0 \qquad (2)$$

$$\sum_i F_{y_i} = 0: \quad A_y - P = 0 \qquad (3)$$

aus (2) $\quad A_x = F_S \qquad (2^*)$

aus (1) $\quad F_S = P \cdot 2l/(h+r) = P \cdot \dfrac{2 \cdot 0.75m}{0.6m}$

$F_S = P \cdot 2.5$

in (2^*) $\quad A_x = 2.5P \quad$, aus (3) $\quad A_y = P$

Betrag: $\quad |A| = \sqrt{A_x^2 + A_y^2} = \sqrt{(2.5P)^2 + P^2} \overset{!}{=} F_{max}$

$$\Rightarrow P = \frac{F_{max}}{\sqrt{7.25}} = \frac{2\,KN}{\sqrt{7.25}} = 0.743\,KN \qquad \underline{P = 743\,N}$$