Bsp. 51:

(1) $$\underline{\underline{\sigma}} = \begin{pmatrix} 18 & 0 & 24 \\ 0 & -50 & 0 \\ 24 & 0 & 32 \end{pmatrix} \text{MPa}$$ aus Symmetrie $\underline{\underline{A}}^T = \underline{\underline{A}}$ $\quad (\underline{\underline{A}}^T = -\underline{\underline{A}})$

Zeile, Spalte
$$\begin{pmatrix} 11 & 12 & 13 \\ 21 & 22 & 23 \\ 31 & 32 & 33 \end{pmatrix}$$

(2) $I_1 = tr(\underline{\underline{\sigma}}) = \sigma_{11} + \sigma_{22} + \sigma_{33} = 18\,\text{MPa} - 50\,\text{MPa} + 32\,\text{MPa} = \underline{0\ \text{N mm}^{-2}}$

$I_2 = \sigma_{12}^2 + \sigma_{13}^2 + \sigma_{23}^2 - \sigma_{11}\sigma_{22} - \sigma_{11}\sigma_{33} - \sigma_{22}\sigma_{33}$ $\quad = \frac{1}{2}\left[tr^2(\underline{\underline{\sigma}}) - tr(\underline{\underline{\sigma}}^2)\right] = \left| \underline{\underline{\sigma}}\cdot\underline{\underline{\sigma}} = \right|$

$= (0)^2 + (24)^2 + (0)^2 - 18\cdot(-50) - 18\cdot 32 - (-50)\cdot 32$ $\quad \frac{1}{2}\left[\sigma_{12}^2 + \sigma_{13}^2 + \sigma_{23}^2 + 2\sigma_{12}\sigma_{13} + 2\sigma_{12}\sigma_{23} + 2\sigma_{12}\sigma_{23} \ldots\right] =$

$= \underline{2500\ \text{N}^2\text{mm}^{-4}}$

$$I_3 = det(\underline{\underline{\sigma}}) = \begin{vmatrix} 18 & 0 & 24 \\ 0 & -50 & 0 \\ 24 & 0 & 32 \end{vmatrix} = 18\cdot(-50)\cdot 32 + 0 + 0 - 24\cdot(-50)\cdot 24 - 0 - 0 = \underline{0\ \text{N}^3\text{mm}^{-6}}$$

$\boxed{\text{singuläre Matrix } \underline{\underline{\sigma}}}$ $\rightarrow$ ∄ Inverse

(3) Hauptspannungen sind die Eigenwerte des Spannungstensors, also ist

$det(\underline{\underline{\sigma}} - \lambda\,\underline{\underline{I}}) = 0$ für $\lambda_1, \lambda_2, \lambda_3$ zu lösen. $\quad$ auch $\lambda^3 - I_1\lambda^2 - I_2\lambda - I_3 = 0$ char. Polynom

$$det\left[\begin{pmatrix} 18 & 0 & 24 \\ 0 & -50 & 0 \\ 24 & 0 & 32 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right] = 0$$

$$det\left[\begin{pmatrix} 18-\lambda & 0 & 24 \\ 0 & -50-\lambda & 0 \\ 24 & 0 & 32-\lambda \end{pmatrix}\right] = 0$$

$(18-\lambda)(-50-\lambda)(32-\lambda) - 24\cdot(-50-\lambda)\cdot 24 = 0$

$[\cancel{576} - 18\lambda - 32\lambda + \lambda^2 - \cancel{576}](-50-\lambda) = 0$ $\quad$ char. Gl.

es gilt: $\quad -50 - \lambda_1 = 0 \quad$ und $\quad \lambda^2 - 50\lambda = 0$

$\underline{\lambda_1 = -50}$ $\quad \lambda(\lambda - 50) = 0$

$\rightarrow \lambda_2 = 0, \quad \lambda_3 = 50$

NB: Suche Vektor $\underline{v}$, sodass $\underline{\underline{A}}\cdot\underline{v} = \lambda\underline{v}$ gilt.

λ .. Eigenwert, $\underline{v}$ .. Eigenvektor

$\Rightarrow (\underline{\underline{A}} - \lambda\underline{\underline{1}})\cdot\underline{v} = \underline{0}$, triviale Lg. $\rightarrow \underline{v} = \underline{0}$

nicht-triviale Lg. für $\underline{v}$ wenn $det(\underline{\underline{A}} - \lambda\underline{\underline{1}}) = 0$ gilt.

$n\times n$-Matrix $\rightarrow$ char. Polynom für $\lambda, \ldots, \lambda^n$

Vorteil in Physik: Trägheitsmomente, Spannungen, Drehungen, Energieniveaus der e^- in Atomhülle, generell QM & QFT

Hauptspannungen (werden absteigend geordnet): $\quad \sigma_1 = 50\,\text{MPa},\ \sigma_2 = 0\,\text{MPa},\ \sigma_3 = -50\,\text{MPa}$

$$\underline{\underline{P}} = \begin{pmatrix} 50 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -50 \end{pmatrix} \text{MPa}$$

(4) $I_{1p} = tr(\underline{\underline{P}}) = 50 + 0 + (-50) = \underline{0} = I_1$ ✓

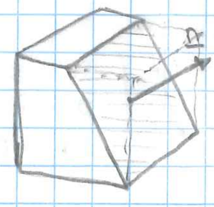
$I_{2p} = \sigma_{12}^2 + \sigma_{13}^2 + \sigma_{23}^2 - \sigma_{11}\sigma_{22} - \sigma_{11}\sigma_{33} - \sigma_{22}\sigma_{33}$

$= -50\cdot(-50) = \underline{2500\ \text{N}^2\text{mm}^{-4}} = I_2$ ✓

$I_{3p} = det(\underline{\underline{P}}) = 0 = I_3$ ✓

Bsp. 52:

$$\underline{\sigma} = \begin{pmatrix} 7 & 0 & -2 \\ 0 & 5 & 0 \\ -2 & 0 & 4 \end{pmatrix} \text{MPa} , \qquad \underline{n} = \begin{pmatrix} 2/3 \\ -2/3 \\ 1/3 \end{pmatrix}$$



(1) Spannungsvektor bei Schnitt:

$$\underline{t}^{(n)} = \underline{\sigma} \cdot \underline{n} = \begin{pmatrix} 7 & 0 & -2 \\ 0 & 5 & 0 \\ -2 & 0 & 4 \end{pmatrix} \cdot \begin{pmatrix} 2/3 \\ -2/3 \\ 1/3 \end{pmatrix} = \begin{pmatrix} \frac{14}{3} - \frac{2}{3} \\ -\frac{10}{3} \\ -\frac{4}{3} + \frac{4}{3} \end{pmatrix} = \underline{\begin{pmatrix} 4 \\ -\frac{10}{3} \\ 0 \end{pmatrix}} \text{ MPa}$$

Projektion auf Schnittfläche

(2) normale Komponente (Normalspannung)

$$\sigma = \underline{t}^{(n)T} \cdot \underline{n} = \left(4 \;\; -\tfrac{10}{3} \;\; 0\right) \cdot \begin{pmatrix} 2/3 \\ -2/3 \\ 1/3 \end{pmatrix} = \frac{8}{3} + \frac{20}{9} + 0 = \frac{24}{9} + \frac{20}{9} = \underline{\frac{44}{9} \text{ MPa}}$$

Betrag der Normalspannung in Schnittfläche

tangentiale Komponente (Schubspannung)

$$\tau^2 = \underline{t}^{(n)2} - \sigma^2 , \qquad \underline{t}^{(n)2} = \underline{t}^{(n)T} \cdot \underline{t}^{(n)} = \left(4 \;\; -\tfrac{10}{3} \;\; 0\right) \cdot \begin{pmatrix} 4 \\ -\frac{10}{3} \\ 0 \end{pmatrix} = 16 + \frac{100}{9}$$

$$\tau^2 = 16 + \frac{100}{9} - \left(\frac{44}{9}\right)^2 = \frac{244}{9} - \frac{1936}{81} = \frac{2196 - 1936}{81} = \frac{260}{81}$$

$$\tau = \sqrt{\frac{260}{81}} = \sqrt{\frac{4 \cdot 65}{81}} = \underline{\frac{2}{9}\sqrt{65} \text{ MPa}}$$

Betrag der Schubspannung in Schnittfläche

Bsp. 53:

(1) $$\underline{\sigma} = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$$

Hauptspannungen: $\det(\underline{\sigma} - \lambda \underline{I}) = 0$

$$\det \begin{pmatrix} 3-\lambda & 1 & 1 \\ 1 & -\lambda & 2 \\ 1 & 2 & -\lambda \end{pmatrix} = 0$$

$$(3-\lambda)\lambda^2 + 2 + 2 + \lambda - 4(3-\lambda) + \lambda = 0$$

$$(3-\lambda)\underbrace{(\lambda^2 - 4)}_{(\lambda-2)(\lambda+2)} + 2(2+\lambda) = 0$$

$$(\lambda+2)\left[(3-\lambda)(\lambda-2) + 2\right] = 0$$

also $(\lambda+2) = 0 \rightarrow \underline{\lambda_1 = -2}$ und $(3-\lambda)(\lambda-2) + 2 = 0$

$$3\lambda - 6 - \lambda^2 + 2\lambda + 2 = 0$$

$$-\lambda^2 + 5\lambda - 4 = 0$$

$$\lambda^2 - 5\lambda + 4 = 0 \qquad \lambda_{i,j} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-b}{2a} \pm \sqrt{\frac{b^2-4ac}{4a^2}} \quad \text{für} \quad ax^2+bx+c=0$$

$$\lambda_{2,3} = \frac{5}{2} \pm \sqrt{\frac{25}{4} - \frac{16}{4}}$$

$$= \frac{5}{2} \pm \sqrt{\frac{9}{4}} = \frac{5}{2} \pm \frac{3}{2} \rightarrow \lambda_2 = \frac{8}{2} = 4, \ \lambda_3 = \frac{2}{2} = \underline{1}$$

niederrum: $\sigma_1 > \sigma_2 > \sigma_3$: $\qquad \sigma_1 = 4, \ \sigma_2 = 1, \ \sigma_3 = -2 \qquad$ (alle MPa)

$$\underline{P} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \text{MPa}$$

(2) Hauptachsenrichtungen:

$\underline{\sigma_1}$: $\qquad (\underline{\sigma} - \sigma_1 \cdot \underline{\underline{I}}) \cdot \underline{n}_1 = \underline{0}$

$$\left[\begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix} - \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_1 & 0 \\ 0 & 0 & \sigma_1 \end{pmatrix}\right] \cdot \begin{pmatrix} n_{1x} \\ n_{1y} \\ n_{1z} \end{pmatrix} = \underline{0}$$

mit $\sigma_1 = 4$: $\quad \begin{pmatrix} -1 & 1 & 1 \\ 1 & -4 & 2 \\ 1 & 2 & -4 \end{pmatrix} \cdot \begin{pmatrix} n_{1x} \\ n_{1y} \\ n_{1z} \end{pmatrix} = \underline{0}$

$$\begin{aligned} -n_{1x} + n_{1y} + n_{1z} &= 0 \quad (\mathrm{I}) \\ n_{1x} - 4n_{1y} + 2n_{1z} &= 0 \quad (\mathrm{II}) \\ n_{1x} + 2n_{1y} - 4n_{1z} &= 0 \quad (\mathrm{III}) \end{aligned}$$

lin. abh.

$$\& \quad n_{1x}^2 + n_{1y}^2 + n_{1z}^2 = 1 \quad (\mathrm{IV})$$

Betrag ist eins!

(II) − (III): $\quad -6n_{1y} + 6n_{1z} = 0 \rightarrow n_{1z} = n_{1y} \quad$ (V)

2(I) − (II): $\quad -3n_{1x} + 6n_{1y} = 0 \rightarrow n_{1x} = 2n_{1y} \quad$ (VI)

(V) & (VI) in (IV): $\quad (2n_{1y})^2 + n_{1y}^2 + n_{1y}^2 = 1$

$$6n_{1y}^2 = 1 \rightarrow n_{1y} = \frac{1}{\sqrt{6}}$$

aus (VI): $\quad n_{1x} = \frac{2}{\sqrt{6}}$

aus (V): $\quad n_{1z} = \frac{1}{\sqrt{6}}$

$$\underline{n_1 = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}}$$

analog für σ_2 & σ_3:

$$(\underline{\sigma} - \sigma_2 \cdot \underline{\underline{I}}) \cdot \underline{n}_2 = 0 \rightarrow \underline{n_2 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}}$$

$$(\underline{\sigma} - \sigma_3 \cdot \underline{\underline{I}}) \cdot \underline{n}_3 = 0 \rightarrow \underline{n_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}}$$

Bsp. 54:

(1) $$\underline{\underline{\sigma}} = \begin{pmatrix} 100 & 20\sqrt{3} & 0 \\ 20\sqrt{3} & 60 & 0 \\ 0 & 0 & 10 \end{pmatrix} N\,mm^{-2}$$

(2) $$\det\left(\underline{\underline{\sigma}} - \lambda \cdot \underline{\underline{I}}\right) = 0$$

$$\begin{vmatrix} 100-\lambda & 20\sqrt{3} & 0 \\ 20\sqrt{3} & 60-\lambda & 0 \\ 0 & 0 & 10-\lambda \end{vmatrix} = 0 \quad \longrightarrow \quad (100-\lambda)(60-\lambda)(10-\lambda) - 1200\,(10-\lambda) = 0$$

$$(10-\lambda)\left[(100-\lambda)(60-\lambda) - 1200\right] = 0 \quad \longrightarrow \quad 10-\lambda = 0 \rightarrow \underline{\lambda_1 = 10}$$

$$6000 - 60\lambda - 100\lambda + \lambda^2 - 1200 = 0$$

$$\lambda^2 - 160\lambda + 4800 = 0$$

$$\lambda_{2,3} = \frac{160}{2} \pm \sqrt{\left(\frac{160}{2}\right)^2 - 4800} = 80 \pm \sqrt{1600} = 80 \pm 40$$

$$\lambda_2 = 120\,, \quad \lambda_3 = 40 \qquad \longrightarrow \qquad \begin{Vmatrix} \sigma_1 = 120\ N\,mm^{-2} \\ \sigma_2 = 40\ N\,mm^{-2} \\ \sigma_3 = 10\ N\,mm^{-2} \end{Vmatrix}$$

(3) Deviator:

$$\underline{\underline{\sigma}} = \underline{\underline{\sigma}}_H + \underline{\underline{\sigma}}' = p \cdot \underline{\underline{I}} + \underline{\underline{\sigma}}' \qquad \text{mit} \quad p = \frac{1}{3}\,tr\left(\underline{\underline{\sigma}}\right) = \frac{1}{3}\left(100 + 60 + 10\right) = \underline{56.\dot{6}\ N\,mm^{-2}}$$

$$\underline{\underline{\sigma}}_H = \begin{pmatrix} 56.\dot{6} & 0 & 0 \\ 0 & 56.\dot{6} & 0 \\ 0 & 0 & 56.\dot{6} \end{pmatrix} N\,mm^{-2}$$

$$\underline{\underline{\sigma}}' = \underline{\underline{\sigma}} - \underline{\underline{\sigma}}_H = \begin{pmatrix} 100 & 20\sqrt{3} & 0 \\ 20\sqrt{3} & 60 & 0 \\ 0 & 0 & 10 \end{pmatrix} - \begin{pmatrix} 56.\dot{6} & 0 & 0 \\ 0 & 56.\dot{6} & 0 \\ 0 & 0 & 56.\dot{6} \end{pmatrix} = \begin{pmatrix} 43.\dot{3} & 20\sqrt{3} & 0 \\ 20\sqrt{3} & 3.\dot{3} & 0 \\ 0 & 0 & -46.\dot{6} \end{pmatrix}$$

(4) Invarianten des Deviators:

$$J_1 = tr\left(\underline{\underline{\sigma}}'\right) = 43.\dot{3} + 3.\dot{3} - 46.\dot{6} = \underline{0}\ \checkmark$$

Deviator = Abw. vom Kugeltensor → gesamte Spur in $\underline{\underline{\sigma}}_H$!

Probe: $I_1 = tr\left(\underline{\underline{\sigma}}\right) = 170\ N\,mm^{-2}$

$$tr\left(\underline{\underline{\sigma}}\right) = tr\left(\underline{\underline{\sigma}}'\right) + tr\left(\underline{\underline{\sigma}}_H\right)$$

$$= J_1 + 3p = 170\ N\,mm^{-2}$$

$$J_2 = \sigma_{12}'^2 + \sigma_{13}'^2 + \sigma_{23}'^2 - \sigma_{11}'\sigma_{22}' - \sigma_{11}'\sigma_{33}' - \sigma_{22}'\sigma_{33}'$$

$$= 20^2 \cdot 3 - 43.\dot{3} \cdot 3.\dot{3} - 43.\dot{3} \cdot (-46.\dot{6}) - 3.\dot{3} \cdot (-46.\dot{6}) = \underline{3228.67\ N^2\,mm^{-4}}$$

$$J_3 = \det\left(\underline{\underline{\sigma}}'\right) = \begin{vmatrix} 43.\dot{3} & 20\sqrt{3} & 0 \\ 20\sqrt{3} & 3.\dot{3} & 0 \\ 0 & 0 & -46.\dot{6} \end{vmatrix} = -6658.7 + 55920 = \underline{49.26 \cdot 10^3\ N^3\,mm^{-6}}$$

Bsp. 55:

(1)
$$\underline{\underline{\sigma}} = \begin{pmatrix} 4 & 4 & 12 \\ 4 & 20 & 4 \\ 12 & 4 & 9 \end{pmatrix} \cdot 10^2 \text{ MPa}$$

(2)
$$p = \frac{1}{3}\, tr(\underline{\underline{\sigma}}) = \frac{1}{3}(40 + 20 + 9) \cdot 10^2 = \underline{11 \cdot 10^2 \text{ MPa}}$$

$$\underline{\underline{\sigma}}_H = p \cdot \underline{\underline{I}} = \underline{\begin{pmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{pmatrix} \cdot 10^2 \text{ MPa}}$$

$$\underline{\underline{\sigma}}' = \begin{pmatrix} 4-11 & 4 & 12 \\ 4 & 20-11 & 4 \\ 12 & 4 & 9-11 \end{pmatrix} \cdot 10^2 \text{ MPa} = \underline{\begin{pmatrix} -7 & 4 & 12 \\ 4 & 9 & 4 \\ 12 & 4 & -2 \end{pmatrix} \cdot 10^2 \text{ MPa}}$$

(3)
$$I_{1H} = tr(\underline{\underline{\sigma}}_H) = \underline{33 \cdot 10^2 \text{ MPa}}$$

$$I_{2H} = \sigma_{H12}^2 + \sigma_{H13}^2 + \sigma_{H23}^2 - \sigma_{11}\sigma_{22} - \sigma_{11}\sigma_{33} - \sigma_{22}\sigma_{33}$$

$$= -3\sigma_{H11}^2 = -3 \cdot (11 \cdot 10^2)^2 = \underline{-3.63 \cdot 10^6 \text{ N}^2\text{mm}^{-4}}$$

$$I_{3H} = det(\underline{\underline{\sigma}}_H) = \underbrace{(11 \cdot 10^2)^3}_{1100^3} = \underline{1.331 \cdot 10^9 \text{ N}^3\text{mm}^{-6}}$$

(4)
$$J_1 = tr(\underline{\underline{\sigma}}') = 0 \quad \text{ist immer der Fall!}$$

$$J_2 = \sigma'^2_{12} + \sigma'^2_{13} + \sigma'^2_{23} - \sigma'_{11}\sigma'_{22} - \sigma'_{11}\sigma'_{33} - \sigma'_{22}\sigma'_{33}$$

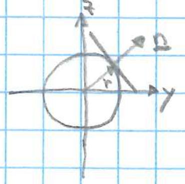
$$= (16 + 144 + 16 + 63 - 14 + 18) 10^4 = \underline{\underline{243 \cdot 10^4 \text{ N}^2\text{mm}^{-4}}}$$

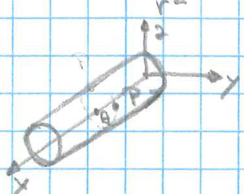
$$J_3 = det(\underline{\underline{\sigma}}') = \begin{vmatrix} -7 & 4 & 12 \\ 4 & 9 & 4 \\ 12 & 4 & -2 \end{vmatrix} 10^2 = (126 + 192 + 192 - 1296 + 112 + 32) 10^6 = \underline{\underline{-642 \cdot 10^6 \text{ N}^3\text{mm}^{-6}}}$$

Bsp. 56:

$$\underline{\underline{\sigma}} = \begin{pmatrix} 6xy & 10y^2 & 0 \\ 10y^2 & 0 & 2z \\ 0 & 2z & 0 \end{pmatrix} 10^1 \text{ MPa}$$

Zylinderfläche: $\underbrace{y^2 + z^2 = 6}_{r^2}$ $\longrightarrow$ Kreisgleichung: $r = \sqrt{6}$





$$P = \begin{pmatrix} 3 \\ 1 \\ \sqrt{5} \end{pmatrix}, \quad A = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$$

$$|\overline{AP}| = \sqrt{1 + 5} = \sqrt{6}$$

$$\underline{n} = \frac{\overline{AP}}{|\overline{AP}|} = \frac{P - A}{|\overline{AP}|} = \frac{\begin{pmatrix} 0 \\ 1 \\ \sqrt{5} \end{pmatrix}}{\sqrt{6}} = \underline{\frac{1}{\sqrt{6}} \begin{pmatrix} 0 \\ 1 \\ \sqrt{5} \end{pmatrix}}$$

Spannungstensor im Punkt P:

$$\underline{\underline{\sigma}}_P = \begin{pmatrix} 6\cdot 3\cdot 1 & 10\cdot 1^2 & 0 \\ 10\cdot 1^2 & 0 & 2\cdot\sqrt{5} \\ 0 & 2\cdot\sqrt{5} & 0 \end{pmatrix} \cdot 10^1\,\mathrm{MPa} = \begin{pmatrix} 180 & 100 & 0 \\ 100 & 0 & 20\cdot\sqrt{5} \\ 0 & 20\sqrt{5} & 0 \end{pmatrix} \mathrm{MPa}$$

$$\underline{t}^{(n)} = \underline{\underline{\sigma}}_P \circ \underline{n} = \begin{pmatrix} 180 & 100 & 0 \\ 100 & 0 & 20\sqrt{5} \\ 0 & 20\sqrt{5} & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ \sqrt{5} \end{pmatrix} \frac{1}{\sqrt{6}}$$

$$= \begin{pmatrix} 100 \\ 100 \\ 20\sqrt{5} \end{pmatrix} \frac{1}{\sqrt{6}} \quad \rightarrow \quad \underline{\underline{t^{(n)} = \frac{100}{\sqrt{6}} \cdot \begin{pmatrix} 1 \\ 1 \\ \sqrt{5}/5 \end{pmatrix} \mathrm{MPa}}}$$

Bsp. 57:

$$\underline{\underline{\sigma}} = \begin{pmatrix} 100 & -100 & 0 \\ -100 & 100 & 0 \\ 0 & 0 & 300 \end{pmatrix} \mathrm{MPa}$$

(1) Normalenvektor: $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ also Einheitsvektor: $\underline{\underline{n = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}}$

$$\underline{t}^{(n)} = \underline{\underline{\sigma}} \cdot \underline{n} = \frac{1}{\sqrt{3}} \begin{pmatrix} 100 & -100 & 0 \\ -100 & 100 & 0 \\ 0 & 0 & 300 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \underline{\frac{1}{\sqrt{3}} \begin{pmatrix} 0 \\ 0 \\ 300 \end{pmatrix} \mathrm{MPa}}$$

(2) Normalkomponente:

$$\sigma = \underline{t}^{(n)T} \cdot \underline{n} = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 & 0 & 300 \end{pmatrix} \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \frac{1}{3} 300 = \underline{\underline{100\,\mathrm{MPa}}}$$

Schubkomponente:

$$\tau^2 = \underline{t}^{(n)2} - \sigma^2 \quad , \quad \underline{t}^{(n)2} = \underline{t}^{(n)T} \cdot \underline{t}^{(n)} = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 & 0 & 300 \end{pmatrix} \frac{1}{\sqrt{3}} \begin{pmatrix} 0 \\ 0 \\ 300 \end{pmatrix}$$

$$= \frac{1}{3} 90\,000 = \underline{\underline{30\,000\,\mathrm{MPa}^2}}$$

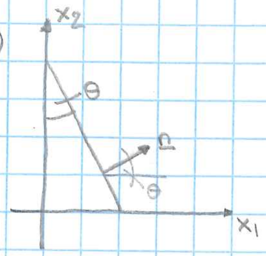
$$\tau^2 = 30\,000 - 10\,000 = 20\,000\,\mathrm{MPa}^2$$

$$\tau = \sqrt{20\,000} = \sqrt{2\cdot 10\,000} = \underline{\underline{100\sqrt{2}\,\mathrm{MPa}}}$$

Bsp. 58:

$$\underline{\underline{\sigma}} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{xy} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$$

(1)



$$n_{x_1} = \cos\theta \qquad n_{x_2} = \sin\theta \quad \rightarrow \quad \underline{n} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}$$

$$\underline{t}^{(n)} = \underline{\underline{\sigma}} \cdot \underline{n} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{xy} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix} \cdot \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} = \underline{\begin{pmatrix} \sigma_{xx}\cos\theta + \sigma_{xy}\sin\theta \\ \sigma_{xy}\cos\theta + \sigma_{yy}\sin\theta \\ 0 \end{pmatrix}}$$

(2) $$\sigma = \underline{t}^{(n)T} \cdot \underline{n} = (\sigma_{xx}\cos\theta + \sigma_{xy}\sin\theta,\ \sigma_{xy}\cos\theta + \sigma_{yy}\sin\theta,\ 0) \cdot \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}$$

$$= \sigma_{xx}\cos^2\theta + \sigma_{xy}\sin\theta\cos\theta + \sigma_{xy}\sin\theta\cos\theta + \sigma_{yy}\sin^2\theta$$

$$= \underline{\sigma_{xx}\cos^2\theta + 2\sigma_{xy}\sin\theta\cos\theta + \sigma_{yy}\sin^2\theta}$$

$$\tau^2 = \underline{t}^{(n)^2} - \sigma^2 \quad ,$$

$$\sigma^2 = (\sigma_{xx}\cos^2\theta + 2\sigma_{xy}\sin\theta\cos\theta + \sigma_{yy}\sin^2\theta)(\sigma_{xx}\cos^2\theta + 2\sigma_{xy}\sin\theta\cos\theta + \sigma_{yy}\sin^2\theta)$$

$$= \sigma_{xx}^2\cos^4\theta + 2\sigma_{xx}\sigma_{xy}\sin\theta\cos^3\theta + \sigma_{xx}\sigma_{yy}\sin^2\theta\cos^2\theta$$

$$+ 2\sigma_{xx}\sigma_{xy}\sin\theta\cos^3\theta + 4\sigma_{xy}^2\sin^2\theta\cos^2\theta + 2\sigma_{xy}\sigma_{yy}\sin^3\theta\cos\theta$$

$$+ \sigma_{xx}\sigma_{yy}\sin^2\theta\cos^2\theta + 2\sigma_{xy}\sigma_{yy}\sin^3\theta\cos\theta + \sigma_{yy}^2\sin^4\theta$$

$$= \sigma_{xx}^2\cos^4\theta + 2\sigma_{xx}\sigma_{yy}\sin^2\theta\cos^2\theta + 4\sigma_{xx}\sigma_{xy}\sin\theta\cos^3\theta + 4\sigma_{xy}^2\sin^2\theta\cos^2\theta$$

$$+ 4\sigma_{xy}\sigma_{yy}\sin^3\theta\cos\theta + \sigma_{yy}^2\sin^4\theta$$

Forb. Bsp 58:

$$\underline{t}^{(n)^2} = \underline{t}^{(n)^T} \cdot \underline{t}^{(n)} = (\sigma_{xx}\cos\theta + \sigma_{xy}\sin\theta,\ \sigma_{xy}\cos\theta + \sigma_{yy}\sin\theta, 0)\begin{pmatrix} \sigma_{xx}\cos\theta + \sigma_{xy}\sin\theta \\ \sigma_{xy}\cos\theta + \sigma_{yy}\sin\theta \\ 0 \end{pmatrix}$$

$$= \sigma_{xx}^2\cos^2\theta + \sigma_{xx}\sigma_{xy}\cos\theta\sin\theta + \sigma_{xy}^2\sin^2\theta + \sigma_{xy}\sigma_{xx}\sin\theta\cos\theta + \sigma_{xy}^2\cos^2\theta + \sigma_{xy}\sigma_{yy}\cos\theta\sin\theta$$

$$+ \sigma_{yy}\sigma_{xy}\sin\theta\cos\theta + \sigma_{yy}^2\sin^2\theta$$

$$= \sigma_{xx}^2\cos^2\theta + 2\sigma_{xx}\sigma_{xy}\sin\theta\cos\theta + \sigma_{xy}^2 + 2\sigma_{xy}\sigma_{yy}\sin\theta\cos\theta + \sigma_{yy}^2\sin^2\theta$$

$$\tau^2 = \underline{t}^{(n)^2} - \sigma^2$$

$$= \sigma_{xx}^2\cos^2\theta + 2\sigma_{xx}\sigma_{xy}\sin\theta\cos\theta + \sigma_{xy}^2 + 2\sigma_{xy}\sigma_{yy}\sin\theta\cos\theta + \sigma_{yy}^2\sin^2\theta - \sigma_{xx}^2\cos^4\theta - 2\sigma_{xx}\sigma_{yy}\sin^2\theta\cos^2\theta$$

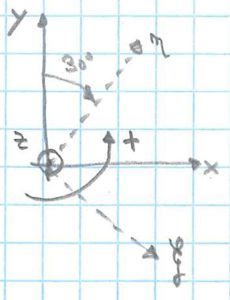
$$-4\sigma_{xx}\sigma_{xy}\sin\theta\cos^3\theta - 4\sigma_{xy}^2\sin^2\theta\cos^2\theta - 4\sigma_{xy}\sigma_{yy}\sin^3\theta\cos\theta - \sigma_{yy}^2\sin^4\theta$$

$$\tau^2 = \sigma_{xx}^2(\cos^2\theta - \cos^4\theta) + \sigma_{yy}^2(\sin^2\theta - \sin^4\theta) + \sigma_{xy}^2(1 - 4\sin^2\theta\cos^2\theta) - 2\sigma_{xx}\sigma_{yy}\sin^2\theta\cos^2\theta$$

$$+ 2\sigma_{xy}\sigma_{yy}(\sin\theta\cos\theta - 2\sin^3\theta\cos\theta) - 2\sigma_{xx}\sigma_{xy}(\sin\theta\cos\theta - 2\sin\theta\cos^3\theta)$$

$$\tau = \sqrt{\tau^2}$$

<u>Bsp. 59:</u> <u>geg.:</u> $\sigma_x = 60\,MPa$
$\sigma_y = -45\,MPa$
$\tau_{xy} = 30\,MPa$



(aktiv vs. passiv)

positiv = gegen Uhrzeigersinn (rechte Hand Regel)

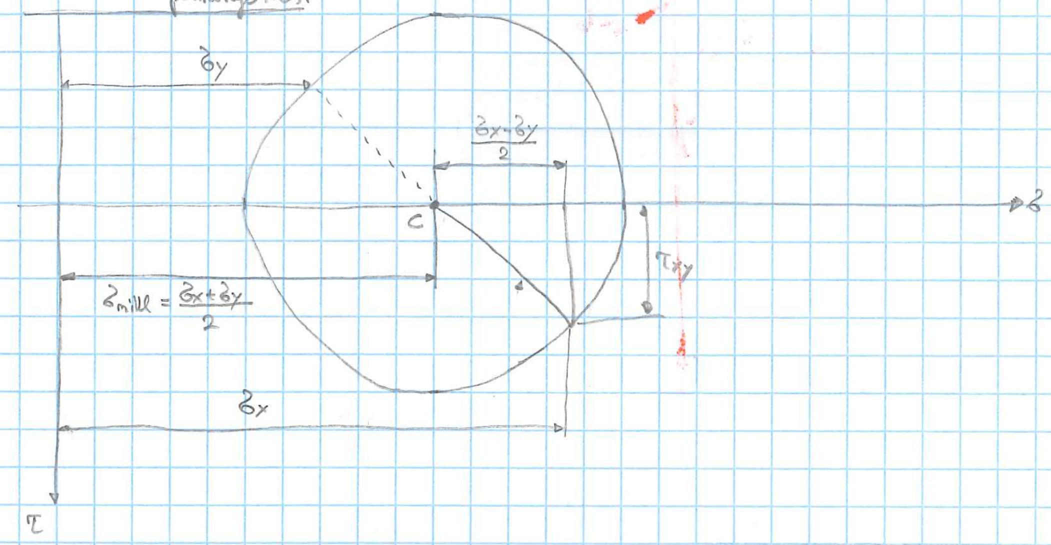
<u>ges.:</u> für $\varphi_A = -30°$ (KS) $\rightarrow \sigma_\xi, \sigma_\eta, \tau_{\xi\eta}$

mittlere Normalspannung: $\sigma_{mittel} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{60\,MPa - 45\,MPa}{2} = \dfrac{15\,MPa}{2} = \underline{7.5\,MPa}$ <u>"Mittelpunkt"</u>

Hauptspannung (des ebenen Spgszustds.): $\tau_{max}^{oder} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\dfrac{60+45}{2}\right)^2 + 30^2}\ MPa$

$= \underline{60.47\,MPa}$ <u>"Radius"</u>

<u>Mohr'scher Spannungskreis:</u>



<u>Richtung der Hauptnormalspannung:</u>

$\tan 2\varphi_0 = \dfrac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} = \dfrac{2 \cdot 30\,MPa}{105\,MPa} = 0.571$

$\underline{2\varphi_0 = 29.73°}$

$\left(\text{oder} \quad \tan \varphi_0 = \dfrac{\tau_{xy}}{\sigma_x - \sigma_{II}} \qquad \tan \varphi_0 = \dfrac{\sigma_I - \sigma_x}{\tau_{xy}} \right)$

<u>NB:</u> $\sigma_{x'} = \dfrac{\sigma_x + \sigma_y}{2} + \dfrac{\sigma_x - \sigma_y}{2} \cos 2\varphi + \tau_{xy} \sin 2\varphi$ Normalspp. in der Ebene ← aus Trigonometrie am Einheitsquadrat des ebenen Spgszustds.

Extremwerte: $\dfrac{d\sigma_{x'}}{d\varphi} \stackrel{!}{=} 0$

$\left.\dfrac{d\sigma_{x'}}{d\varphi}\right|_{\varphi_0} = \dfrac{\sigma_x - \sigma_y}{2} \cdot (-\sin 2\varphi_0) \cdot \cancel{2} + \tau_{xy} \cos 2\varphi_0 \cdot \cancel{2} = 0$

$\tau_{xy} \cos 2\varphi_0 = \dfrac{\sigma_x - \sigma_y}{2} \sin 2\varphi_0$

$\tan 2\varphi_0 = \dfrac{\sin 2\varphi_0}{\cos 2\varphi_0} = \dfrac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}}$ ✓

Von Hauptnormalspannungslage zu 30° gedrehten Lage:

$2\varphi_{neu} = 2\varphi_0 + 2\varphi_A = 29.73° + 2 \cdot 30° = \underline{89.73°}$

Neue Spannungen:

$$\sigma_\xi = \sigma_{mitte} + R \cdot \cos 2\varphi_{neu}$$

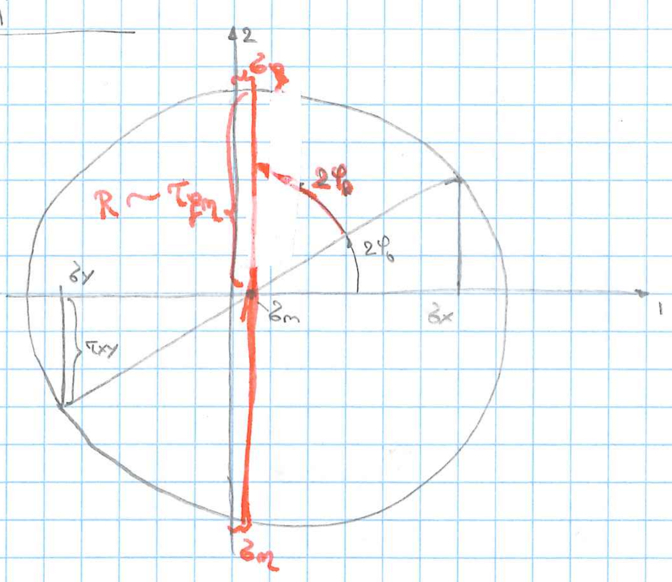
$$\sigma_\xi = 7.5\,MPa + 60.47\,MPa \cdot \cos 89.74^\circ = \underline{7.77\,MPa}$$

$$\sigma_\eta = \sigma_{mitte} - R \cdot \cos 2\varphi_{neu}$$

$$\sigma_\eta = 7.5\,MPa - 60.47\,MPa \cdot \cos 89.74^\circ = \underline{7.22\,MPa}$$

$$\tau_{\xi\eta} = R \cdot \underbrace{\sin 2\varphi_{neu}}_{\sim 1} = \underline{60.47\,MPa}$$

Zeichnerisch (skizziert):



Bsp. 60: $\sigma_x = -15\,MPa$, $\sigma_y = 10\,MPa$, $\tau_{xy} = 8\,MPa$ ges.: $\sigma_{1,2}$, φ_0, τ_{max} Theorieblatt!

(1) Rechnerisch:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{-5}{2} \pm \sqrt{156.25 + 64} = \left(-2.5 \pm \sqrt{220.25}\right) MPa$$

$$\boxed{\sigma_1 = 12.34\,MPa \quad \sigma_2 = -17.34\,MPa}$$

$$\tan 2\varphi_0 = \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)} = \frac{2 \cdot 8\,MPa}{-15\,MPa - 10\,MPa} = -0.64 \longrightarrow 2\varphi_0 = -32.62^\circ$$

$$\boxed{\varphi_0 = -16.30^\circ}$$

$$\tau_{max}^{eben} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \stackrel{!}{=} \frac{\sigma_1 - \sigma_2}{2}$$

Beweis:

$$\sigma_1 - \sigma_2 = \cancel{\frac{\sigma_x + \sigma_y}{2}} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} - \left(\cancel{\frac{\sigma_x + \sigma_y}{2}} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\right)$$

, $\sqrt{\ } > 0$ immer, wg. Quadrieren

d.h. σ_1 ist immer ⊕

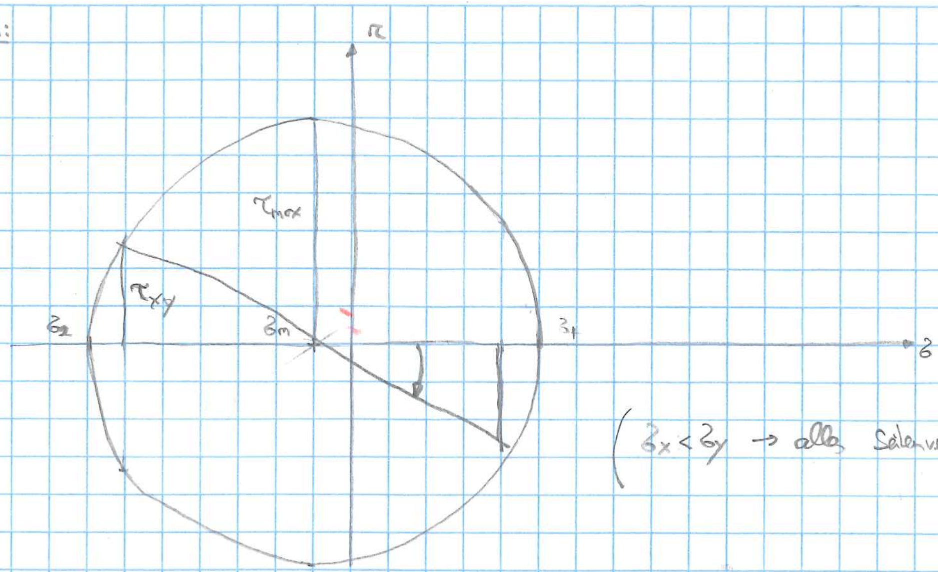
σ_2 ist immer ⊖

$$= 2\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\rightarrow \frac{\sigma_1 - \sigma_2}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

q.e.d., also $\tau_{max}^{eben} = \frac{\sigma_1 - \sigma_2}{2} = \frac{12.34 + 17.34}{2} = \underline{14.84\,MPa}$

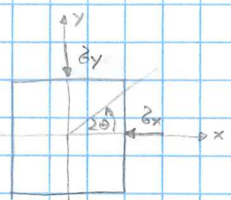
(2) Graphisch:



$\left(\sigma_x < \sigma_y \rightarrow \text{alles Seitenverkehrt!} \right)$

Bsp. 61: Ebener Spannungszustand, $\sigma_x = -80\,MPa$, $\sigma_y = -20\,MPa$ hier $\sigma_x = \sigma_1$, $\sigma_y = \sigma_2$

(1) Schnitt $\theta = 22.5^\circ$ zur x-Achse



$$\sigma_\xi = \sigma_M + R \cdot \cos 2\theta = \left(\frac{\sigma_1 + \sigma_2}{2}\right) + \left(\frac{\sigma_1 - \sigma_2}{2}\right) \cos 45^\circ$$

$$= \frac{-80 - 20}{2} + \frac{-80 + 20}{2} \cos 45^\circ = -50 - 21.21 = \underline{-71.21\,MPa}$$

$$\sigma_\eta = \sigma_M - R \cdot \cos 2\theta = -50 + 21.21 = \underline{-28.79\,MPa}$$

$$\tau_{\xi\eta} = R \cdot \sin 2\theta = \underline{-21.21\,MPa} \quad \longrightarrow \text{ klar, weil } \cos(45^\circ) = \sin(45^\circ) \;!$$

(2) Maximale Schubspannung:

$$\tau_{max}^{abs} = \frac{\sigma_1 - \sigma_2}{2} = \frac{-80 + 20}{2} = \underline{-30\,MPa} = R, \quad \text{Richtung: } 2\theta + 2\theta_{\tau max} = 90^\circ$$

$$\sigma_m = \frac{\sigma_1 + \sigma_2}{2} = \frac{-80 - 20}{2} = \underline{-50\,MPa} \qquad \theta_{\tau max} = \frac{90^\circ - 2\theta}{2} = \underline{22.5^\circ}$$

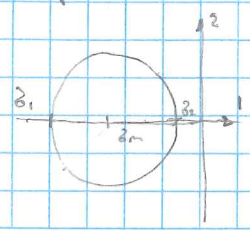
Richtung der maximalen Schubspannung

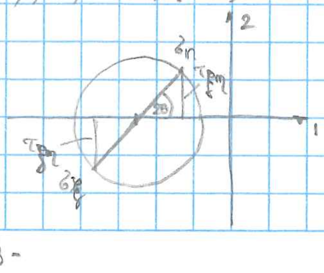
Zeichnerischer Arbeitsplan (ges.: σ_x, σ_y für $\theta = 22.5^\circ$)

(1) Zeichne σ_1 & σ_2 ein (σ_x & σ_y), Mittelpunkt $= \frac{\overline{\sigma_1 \sigma_2}}{2}$ ($\frac{\overline{\sigma_x \sigma_y}}{2}$), Radius $r = \overline{\sigma_M \sigma_1}$

(2) Zeichne 2θ im 1. Quadranten gegen UZS ein (⊕ Richtung = GUZS)

(3) Schnittpunkte der Geraden 2θ sind $\sigma_x, \sigma_y, \tau_{xy}$ (auf jeweiligen Achsen abgelesen).

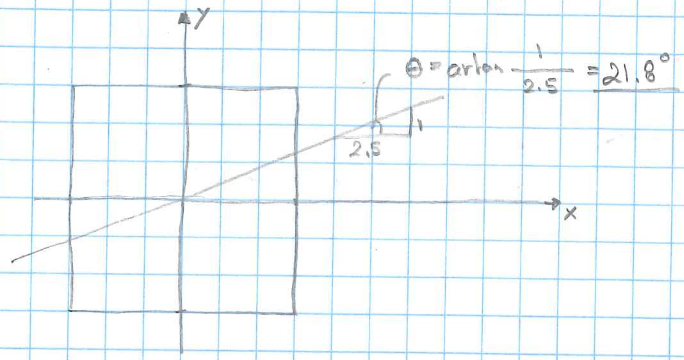




Bsp. 62: Ebener Spannungszustand $\sigma_x = -16\,MPa$
$\sigma_y = 42\,MPa$

$\tau_{xy} = 0$, also gilt: $\sigma_1 = \sigma_x = -16\,MPa$
$\sigma_2 = \sigma_y = 42\,MPa$

(1)



$$\sigma_{\xi\eta} = \frac{\sigma_1 + \sigma_2}{2} \pm \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta = \frac{-16+42}{2} \pm \frac{-16-42}{2} \cos(2 \cdot 21.8°)$$

$$= 13 \pm (-21)$$

$\sigma_\xi = -8\,MPa$, $\sigma_\eta = 34\,MPa$

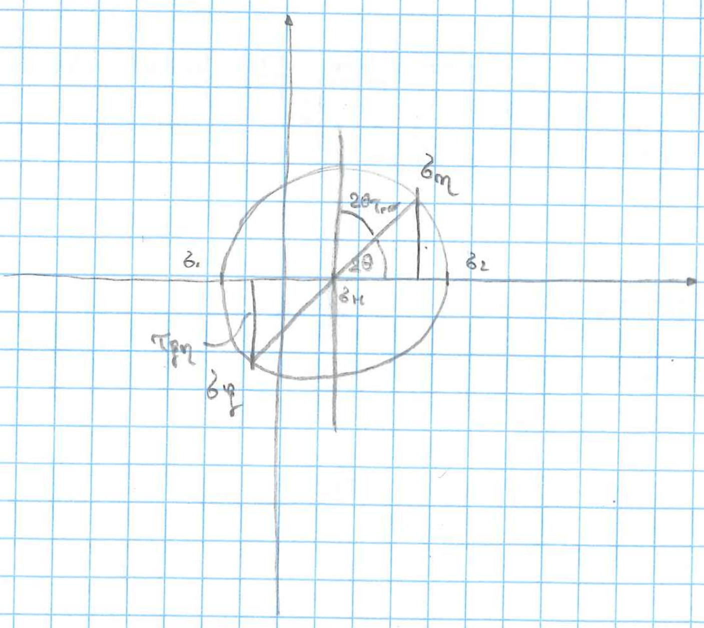
$$\tau_{\xi\eta} = \frac{\sigma_1 - \sigma_2}{2} \sin(2\theta) = \frac{-16-42}{2} \sin(43.6) = -20\,MPa$$

(2) Maximale Schubspg.:

$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = -29\,MPa$, Richtung: $2\theta + 2\theta_{\tau_{max}} = 90° \rightarrow \theta_{\tau_{max}} = \frac{90° - 2\theta}{2}$

$\sigma_m = \frac{\sigma_1 + \sigma_2}{2} = \frac{-16+42}{2} = 13\,MPa$ $\qquad \theta_{\tau_{max}} = 23.2°$

Zeichnerisch:



Bsp. 62:

$$\underline{v}(x,y,z) = \begin{pmatrix} u \\ v \\ w \end{pmatrix} = k \begin{pmatrix} x \\ y + 4z \\ 4\sqrt{2}\,x + 3z \end{pmatrix}$$

(1) $$\underline{\underline{\varepsilon}} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix}$$ oft auch $\varepsilon_{xx} = \varepsilon_x$, $\varepsilon_{yy} = \varepsilon_y$, $\varepsilon_{zz} = \varepsilon_z$

⚠ Symmetrie siehe Ableitung!

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = \frac{\partial (kx)}{\partial x} = \underline{k}$$

d.h. Ableitung der x-Komponente von $\underline{v}$ (Verschiebungsfeld) nach x
→ also deren Änderung bei Variation der x-Koordinate!

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} = \frac{\partial (ky + 4kz)}{\partial y} = \underline{k}$$

$$\varepsilon_{zz} = \frac{\partial w}{\partial z} = \frac{\partial (4k\sqrt{2}x + 3kz)}{\partial z} = \underline{3k}$$

$$\varepsilon_{xy} = \frac{1}{2}\gamma_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) = \varepsilon_{yx} \quad !$$

also x nach y & y nach x

also $\varepsilon_{ij} = \varepsilon_{ji}$ symmetrisch

$$\varepsilon_{xy} = \frac{1}{2}(0+0) = \underline{0} = \varepsilon_{yx}$$

$$\varepsilon_{yz} = \frac{1}{2}\gamma_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = \frac{1}{2}(4k + 0) = \underline{2k} = \varepsilon_{zy}$$

$$\varepsilon_{xz} = \frac{1}{2}\gamma_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = \frac{1}{2}(0 + 4k\sqrt{2}) = \underline{2k\sqrt{2}} = \varepsilon_{zx}$$

Symmetrie setzt sich fort auf $\underline{\underline{\sigma}}$!

also ist der Verzerrungstensor

$$\underline{\underline{\varepsilon}} = \begin{pmatrix} k & 0 & 2k\sqrt{2} \\ 0 & k & 2k \\ 2k\sqrt{2} & 2k & 3k \end{pmatrix} = k \begin{pmatrix} 1 & 0 & 2\sqrt{2} \\ 0 & 1 & 2 \\ 2\sqrt{2} & 2 & 3 \end{pmatrix}$$

(2) Hauptdehnungen

Analog zu Hauptspannungen → Eigenwerte des Verzerrungstensors!

$$\det(\underline{\underline{\varepsilon}} - \lambda \cdot \underline{\underline{I}}) = 0$$

k nachher multiplizieren!

$$\begin{vmatrix} 1-\lambda & 0 & 2\sqrt{2} \\ 0 & 1-\lambda & 2 \\ 2\sqrt{2} & 2 & 3-\lambda \end{vmatrix} = \left[(1-\lambda)^2(3-\lambda) - (1-\lambda)(2\sqrt{2})^2 - 4(1-\lambda)\right] = (1-\lambda)\left[(1-\lambda)(3-\lambda) - 8 - 4\right]$$

$$= (1-\lambda)\left[3 - \lambda - 3\lambda + \lambda^2 - 12\right] = k(1-\lambda)\left[\lambda^2 - 4\lambda - 9\right]$$

$$(1-\lambda) = 0 \rightarrow \lambda_1 = 1$$

$$x_{1,2} = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\lambda^2 - 4\lambda - 9 = 0 \rightarrow \lambda_{2,3} = \frac{4}{2} \pm \frac{\sqrt{16 - 4(-9)}}{2}$$

$$= 2 \pm \frac{\sqrt{4 \cdot 13}}{2} = 2 \pm \sqrt{13} \rightarrow \lambda_2 = 2 + \sqrt{13}, \ \lambda_3 = 2 - \sqrt{13}$$

absteigend geordnet

$$\varepsilon_1 = (2+\sqrt{13})k$$
$$\varepsilon_2 = k$$
$$\varepsilon_3 = (2-\sqrt{13})k$$

$$\underline{\underline{\varepsilon}}^H = \begin{pmatrix} 2+\sqrt{13} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2-\sqrt{13} \end{pmatrix} k$$

Hauptlage

(3) Volumendilatation: $\frac{\Delta V}{V} = tr(\underline{\underline{\varepsilon}})$ 1. Invariante!

also entweder $tr(\underline{\underline{\varepsilon}}'') = (2 + \cancel{\sqrt{13}} + 1 + 2 - \cancel{\sqrt{13}})K = \underline{5K}$

oder $tr(\underline{\underline{\varepsilon}}) = (1 + 1 + 3)K = \underline{5K}$

Bsp. 64: Bauteildehnungen:

$\varepsilon_0 = 0.6 \cdot 10^{-3}$ $\theta_a = 0°$
$\varepsilon_{90} = 1.8 \cdot 10^{-3}$ $\theta_b = 90°$ zur x-Achse
$\varepsilon_{225} = -1.1 \cdot 10^{-3}$ $\theta_c = 225°$

ges.: Hauptdehnungen & deren Richtung

3 Gleichungen aus Hauptgl. für die Dehnung (Herleitung: Verlängerung des Linienelements.)

$$\varepsilon_{x'} = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$$

Für DMS bedeutet das:

$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy} \sin\theta_a \cos\theta_a = \varepsilon_0 \qquad (1)$$

$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy} \sin\theta_b \cos\theta_b = \varepsilon_{45} \qquad (2)$$

$$\varepsilon_c = \varepsilon_x \cos^2\theta_c + \varepsilon_y \sin^2\theta_c + \gamma_{xy} \sin\theta_c \cos\theta_c = \varepsilon_{90} \qquad (3)$$

aus (1) mit $\sin 0 = 0$, $\cos 0 = 1$ folgt $\varepsilon_0 = \varepsilon_x$ (4)

aus (2) mit $\sin 90° = 1$, $\cos 90° = 0$ folgt $\varepsilon_{90} = \varepsilon_y$ (5)

aus (3) mit $\cos(225) = 1/\sqrt{2}$, $\sin(225) = 1/\sqrt{2}$ folgt $\varepsilon_{225} = \frac{1}{2}\varepsilon_x + \frac{1}{2}\varepsilon_y + \frac{1}{2}\gamma_{xy} \rightarrow \frac{1}{2}\gamma_{xy} = \varepsilon_{225} - \frac{1}{2}\varepsilon_0 - \frac{1}{2}\varepsilon_{90}$

Hauptdehnungen:

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \frac{\varepsilon_0 + \varepsilon_{90}}{2} \pm \sqrt{\left(\frac{\varepsilon_0 - \varepsilon_{90}}{2}\right)^2 + \left(\varepsilon_{225} - \frac{1}{2}\varepsilon_0 - \frac{1}{2}\varepsilon_{90}\right)^2}$$

$$= 1.2 \cdot 10^{-3} \pm 2.38 \cdot 10^{-3}$$

$\underline{\varepsilon_1 = 3.58 \cdot 10^{-3}}$, $\underline{\varepsilon_2 = -1.18 \cdot 10^{-3}}$

Richtung: $\tan 2\varphi_0 = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{2\varepsilon_{225} - \varepsilon_0 - \varepsilon_{90}}{\varepsilon_0 - \varepsilon_{90}} = 3.83$

$2\varphi_0 = \arctan 3.83 = 75.4° \rightarrow \boxed{\varphi_0 = 37.7°}$

Bsp. 65:

Materialpunkt mit ebenem Dehnungszustand

$\varepsilon_x = 480 \cdot 10^{-6}$
$\varepsilon_y = 140 \cdot 10^{-6}$
$\gamma_{xy} = -350 \cdot 10^{-6}$

(1)

$$\underline{\underline{\varepsilon}} = \begin{pmatrix} 480 & -350 & 0 \\ -350 & 140 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot 10^{-6}$$

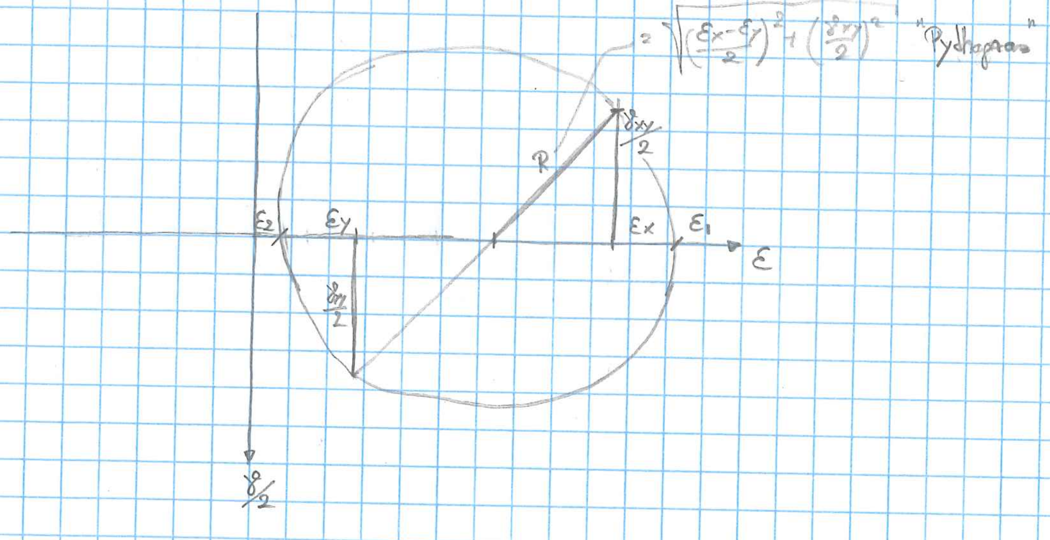
Analog zu den Hauptspannungen gilt:

$$\tan 2\theta_H = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-350}{480-140} = -1.03 \rightarrow \underline{\theta_H = -22.92^\circ} \quad \text{Richtung}$$

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 310 \cdot 10^{-6} \pm 244 \cdot 10^{-6}$$

$$\underline{\varepsilon_1 = 554 \cdot 10^{-6}}, \quad \underline{\varepsilon_2 = 66 \cdot 10^{-6}}$$

$$\underline{\gamma^{abs}_{max}} = 2\sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 2 \cdot 244 \cdot 10^{-6} = \underline{488 \cdot 10^{-6}}$$



Hooke'sches Gesetz: $\underline{\sigma} = \underline{\underline{\underline{\underline{E}}}} : \underline{\underline{\varepsilon}}$, $\sigma_{ij} = E_{ijkl}\,\varepsilon_{kl} = \sum_{k,l} E_{ijkl}\,\varepsilon_{kl}$ / von Feder $\left.\begin{array}{l}\sigma = \frac{F}{A}\\ \varepsilon = \frac{\Delta l}{l_0}\end{array}\right\}$ $\frac{F}{A} = E\,\frac{\Delta l}{l_0} \to F = \frac{E \cdot A}{l_0}\,\Delta l$
$= \underline{K \cdot \Delta l}$ (Feder)

$\underline{\sigma} = \underline{\underline{\underline{\underline{E}}}} : \underline{\underline{\varepsilon}}$

Poisson-Zahl (Querzahl), Wärmeausdehnungs-Koeff.

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] + \alpha_T\,\Delta T \;, \quad \gamma_{xy} = \frac{1}{G}\tau_{xy}$$
$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] + \alpha_T\,\Delta T \;, \quad \gamma_{yz} = \frac{1}{G}\tau_{yz}$$
$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] + \alpha_T\,\Delta T \;, \quad \gamma_{zx} = \frac{1}{G}\tau_{zx}$$

E allg. 4. Stufe: 81 Komp.
Symmetrie von $\sigma, \varepsilon \to$ 36 Komp.
$C_{ijkl} \to E_{IJ}$, $\to$ symmetrisch
max. 21 d. Grad. $\to$ isotrop 2: E, ν

$G = \frac{E}{2(1+\nu)}$, Schubmodul

Bsp. 66: RB $\sigma_x = \sigma_z = 0$
Verschiebung $\Delta a = 0.1\,mm$ $\to$ $\underline{\varepsilon_y} = -\frac{\Delta a}{a} = -\frac{0.1\,mm}{1000\,mm} = \underline{-1 \cdot 10^{-4}}$
Druck

Hooke

$$\varepsilon_x = \frac{1}{E}\left[\overset{=0}{\sigma_x} - \nu(\sigma_y + \overset{=0}{\sigma_z})\right] + \underbrace{\alpha_T\,\Delta T}_{=0} = -\frac{1}{E}\nu\sigma_y \to \varepsilon_x = -\frac{1}{E}\nu\sigma_y \quad (1)$$
$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\underbrace{(\sigma_x + \sigma_z)}_{=0\ (RB)}\right] + \underbrace{\alpha_T\,\Delta T}_{=0,\ da\ \Delta T = 0} = \frac{1}{E}\sigma_y \to \varepsilon_y = \frac{1}{E}\sigma_y \quad (2)$$
$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] + \alpha_T\,\Delta T = -\frac{1}{E}\nu\sigma_y \to \varepsilon_z = -\frac{1}{E}\nu\sigma_y \quad (3)$$

3 unbekannte $\varepsilon_x, \varepsilon_z, \sigma_y$
3 Gl. ✓

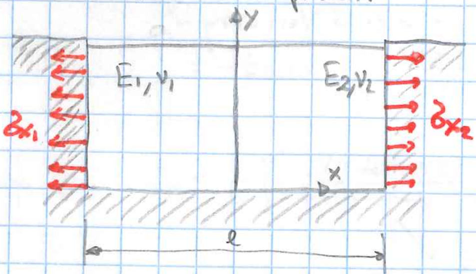
aus (2) $\sigma_y = E\,\varepsilon_y = 210.000\,Nmm^{-2} \cdot (-1 \cdot 10^{-4}) = \underline{-21\,Nmm^{-2}}$

aus (1) $\underline{\varepsilon_x} = -\frac{1}{E}\nu\sigma_y = -\frac{1}{210\,000\,Nmm^{-2}} \cdot 0.3 \cdot (-21\,Nmm^{-2}) = \underline{3 \cdot 10^{-5}}$

aus (3) $\varepsilon_z = -\frac{1}{E}\nu\sigma_y = \varepsilon_x = \underline{3 \cdot 10^{-5}}$

Bsp. 67:

nach dem Einpressen: ges.: σ, Δ



$\sigma_{x_1} = \sigma_{x_2} = \sigma_x$
$\sigma_y = 0$ (weil in y frei) , $\sigma_z = 0$ ebener Spannungszustand (vorne & hinten frei)

Hooke'sches Gesetz: Scheibe 1: $\varepsilon_{x_1} = \frac{1}{E_1}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] = \frac{1}{E_1}\sigma_x$

Scheibe 2: $\varepsilon_{x_2} = \frac{1}{E_2}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] = \frac{1}{E_2}\sigma_x$

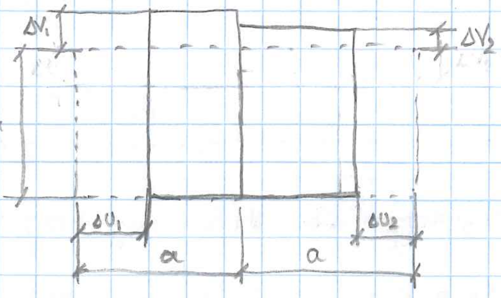
und auch Scheibe 1: $\varepsilon_{y_1} = \frac{1}{E_1}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] = -\frac{1}{E_1}\nu\sigma_x$

Scheibe 2: $\varepsilon_{y_2} = \frac{1}{E_2}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] = -\frac{1}{E_2}\nu\sigma_x$

Zusammenhang Dehnung & Änderung der Seitenlänge:

$$\varepsilon_{x_1} = \frac{|\Delta U_1|}{a}, \qquad \varepsilon_{y_1} = \frac{|\Delta V_1|}{a}$$

$$\varepsilon_{x_2} = \frac{|\Delta U_2|}{a}, \qquad \varepsilon_{y_2} = \frac{|\Delta V_2|}{a}$$



$\Delta U_1, \Delta U_2$ & $\Delta V_1, \Delta V_2$ aufgrund unterschiedliche E-Moduli verschieden

Kinematische Verträglichkeit:

$$(a + |\Delta U_1|) + (a + |\Delta U_2|) = l$$

mit obigem Zusammenhang $\Longrightarrow$ $a + \varepsilon_{x_1} \cdot a + a + \varepsilon_{x_2} \cdot a = l$

$$\varepsilon_{x_1} \cdot a + \varepsilon_{x_2} \cdot a = l - 2a$$

mit $\varepsilon_{x_1} = \frac{1}{E_1} \sigma_x$, $\varepsilon_{x_2} = \frac{1}{E_2} \sigma_x$ folgt

$$\frac{1}{E_1} \sigma_x \cdot a + \frac{1}{E_2} \sigma_x \cdot a = l - 2a$$

$$\sigma_x \cdot a \left(\frac{1}{E_1} + \frac{1}{E_2} \right) = l - 2a \quad \Longrightarrow \quad \sigma_x = \frac{l-2a}{a} \left(\frac{1}{\frac{1}{E_1} + \frac{1}{E_2}} \right) = \frac{l-2a}{a} \left(\frac{E_1 E_2}{E_1 + E_2} \right)$$

$$\boxed{\sigma_x = \frac{l-2a}{a} \cdot \frac{E_1 E_2}{E_1 + E_2}}$$

mit $l < 2a \rightarrow l - 2a < 0 \rightarrow \sigma_x < 0$

$\rightarrow$ Druckspg. ✓

Änderung der Seitenlängen:

$$\Delta U_1 = \varepsilon_{x_1} \cdot a = \frac{\sigma_x}{E_1} a = \frac{\cancel{a}}{\cancel{E_1}} \frac{l-2a}{\cancel{a}} \frac{\cancel{E_1} E_2}{E_1 + E_2}$$

$$\boxed{\Delta U_1 = (l-2a) \frac{E_2}{E_1 + E_2}}$$ mit $l - 2a < 0 \rightarrow \Delta U_1 < 0 \rightarrow$ Verkürzung ✓

$$\Delta U_2 = \varepsilon_{x_2} \cdot a = \frac{\sigma_x}{E_2} \cdot a = \frac{\cancel{a}}{\cancel{E_2}} \frac{l-2a}{\cancel{a}} \frac{E_1 \cancel{E_2}}{E_1 + E_2}$$

$$\boxed{\Delta U_2 = (l-2a) \frac{E_1}{E_1 + E_2}}$$ auch hier Verkürzung ✓

Querdehnung:

$$\varepsilon_{y_1} = -\frac{1}{E_1} \nu \sigma_x = -\frac{1}{\cancel{E_1}} \nu \frac{l-2a}{a} \frac{\cancel{E_1} E_2}{E_1 + E_2} = -\frac{(l-2a)\nu}{a} \frac{E_2}{E_1 + E_2}$$

$$\boxed{\Delta V_1 = \varepsilon_{y_1} \cdot a = -(l-2a)\nu \frac{E_2}{E_1 + E_2}}$$ $l - 2a < 0 \rightarrow$ Verlängerung ✓

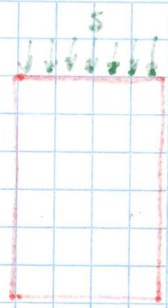
$$\varepsilon_{y_2} = -\frac{1}{E_2} \nu \sigma_x = -\frac{1}{\cancel{E_2}} \nu \frac{l-2a}{a} \frac{E_1 \cancel{E_2}}{E_1 + E_2} = -\frac{(l-2a)\nu}{a} \frac{E_2}{E_1 + E_2} \rightarrow \boxed{\Delta V_2 = -(l-2a)\nu \frac{E_2}{E_1 + E_2}}$$

$\rightarrow$ Verlängerung ✓

Bsp 68:

RB: $\sigma_y = 0$, $\varepsilon_x = \varepsilon_z = 0$

weil vorne frei — weil Führungsstück völlig starr



also gilt mit allg. Hooke'schen Gesetz:

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\,(\underbrace{\cancel{\sigma_y}}_{=0} + \sigma_z)\right] + \alpha_T\,\Delta T \overset{!}{=} 0$$

$$\varepsilon_y = \frac{1}{E}\left[\underbrace{\cancel{\sigma_y}}_{=0} - \nu\,(\sigma_x + \sigma_z)\right] + \alpha_T\,\Delta T$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\,(\sigma_x + \underbrace{\cancel{\sigma_y}}_{=0})\right] + \alpha_T\,\Delta T \overset{!}{=} 0$$

d.h. weiters:

$$\frac{1}{E}\left[\sigma_x - \nu\,\sigma_z\right] + \alpha_T\,\Delta T = 0 \qquad (1)$$

$$\frac{1}{E}\left[-\nu\,(\sigma_x + \sigma_z)\right] + \alpha_T\,\Delta T = \varepsilon_y \qquad (2)$$

$$\frac{1}{E}\left[\sigma_z - \nu\,\sigma_x\right] + \alpha_T\,\Delta T = 0 \qquad (3)$$

3 Unbekannte: $\sigma_x, \sigma_z, \varepsilon_y$; 3 Glg. ✓

aus (1): $\frac{1}{E}\sigma_x = \frac{\nu}{E}\sigma_z - \alpha_T\,\Delta T \rightarrow \sigma_x = \nu\,\sigma_z - E\,\alpha_T\,\Delta T \quad (1^*)$

aus (3): $\frac{1}{E}\sigma_z = \frac{\nu}{E}\sigma_x - \alpha_T\,\Delta T \rightarrow \sigma_z = \nu\,\sigma_x - E\,\alpha_T\,\Delta T \quad (3^*)$

(3^*) in (1^*):

$$\sigma_x = \nu\,(\nu\,\sigma_x - E\,\alpha_T\,\Delta T) - E\,\alpha_T\,\Delta T$$

$$\sigma_x\,(1-\nu^2) = -E\,\alpha_T\,\Delta T\,(1+\nu)$$

$$\sigma_x = -\frac{E\,\alpha_T\,\Delta T\,(1+\nu)}{(1-\nu^2)} = -\frac{E\,\alpha_T\,\Delta T\,\cancel{(1+\nu)}}{\cancel{(1+\nu)}\,(1-\nu)} \longrightarrow \underline{\sigma_x} = -\frac{E\,\alpha_T\,\Delta T}{1-\nu} = \underline{-123.6\text{ MPa}}$$

Druckspg. ✓

mit (3^*):

$$\underline{\sigma_z} = -\nu\,\frac{E\,\alpha_T\,\Delta T}{1-\nu} - E\,\alpha_T\,\Delta T = -E\,\alpha_T\,\Delta T\left(\frac{\nu}{1-\nu} + 1\right)$$

$$= -E\,\alpha_T\,\Delta T\left(\frac{\cancel{\nu} + 1 - \cancel{\nu}}{1-\nu}\right) = -\frac{E\,\alpha_T\,\Delta T}{1-\nu} = \sigma_x = \underline{-123.6\text{ MPa}} \quad ✓$$

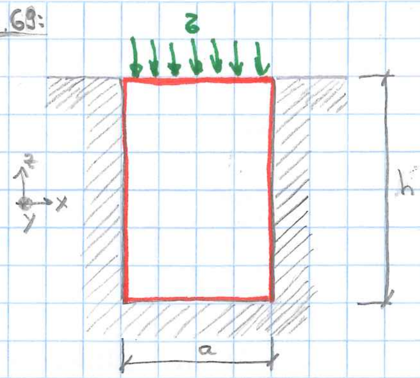
aus (2)

$$\underline{\varepsilon_y} = \frac{1}{\cancel{E}}\left[-\nu\left(-\frac{2\cancel{E}\,\alpha_T\,\Delta T}{1-\nu}\right)\right] + \alpha_T\,\Delta T = \nu\,\frac{2\,\alpha_T\,\Delta T}{1-\nu} + \alpha_T\,\Delta T$$

$$= \alpha_T\,\Delta T\left(\frac{2\nu}{1-\nu} + 1\right) = \alpha_T\,\Delta T\left(\frac{\cancel{2}\nu + 1 - \cancel{\nu}}{1-\nu}\right) = \alpha_T\,\Delta T\,\frac{1+\nu}{1-\nu} = \underline{7.8\cdot 10^{-4}}$$

$[K^{-1}\cdot K] = [1]$ ✓

Bsp. 69:



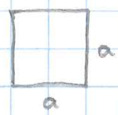
$a = 40\,\text{mm}$

$h = 60\,\text{mm}$

$E = 2.3 \cdot 10^5\,\text{MPa}$

$\alpha_T = 1.2 \cdot 10^{-5}\,\text{K}^{-1}$

$\nu = 0.3$



(1) Δh für Druckbelastung $F = 160\,\text{kN}$ von oben

$$\sigma_z = -\frac{F}{A} \left(= -\frac{160 \cdot 10^3\,\text{N}}{(40\,\text{mm})^2} = \underline{-100\,\text{MPa}} \right)$$

RB: $\varepsilon_{xx} = \varepsilon_{yy} = 0$

Hooke'sches Gesetz (ohne Temperaturterm, da $\Delta T = 0$)

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] \stackrel{!}{=} 0 \qquad (1)$$

$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] \stackrel{!}{=} 0 \qquad (2)$$

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] = \frac{\Delta h}{h} \quad \left(\text{aus } \varepsilon_z = \frac{dw}{dz}\right) \qquad (3)$$

aus (1) $\frac{1}{E}\sigma_x = \frac{\nu}{E}(\sigma_y + \sigma_z) \rightarrow \sigma_x = \nu(\sigma_y + \sigma_z) \qquad (1^*)$

aus (2) $\frac{1}{E}\sigma_y = \frac{\nu}{E}(\sigma_x + \sigma_z) \rightarrow \sigma_y = \nu(\sigma_x + \sigma_z) \qquad (2^*)$

(2^*) in (1^*): $\sigma_x = \nu(\nu\sigma_x + \nu\sigma_z + \sigma_z) = \nu^2\sigma_x + \nu^2\sigma_z + \nu\sigma_z$

$$\sigma_x(1-\nu^2) = \nu\sigma_z(1+\nu) \rightarrow \sigma_x = -\frac{F}{A}\frac{\nu(1+\nu)}{1-\nu^2} = -\frac{F}{A}\frac{\nu\cancel{(1+\nu)}}{\cancel{(1+\nu)}(1-\nu)}$$

$$\underline{\sigma_x = -\frac{F}{A}\frac{\nu}{1-\nu}}$$

aus (2^*)

$$\underline{\sigma_y} = \nu\sigma_x + \nu\sigma_z = -\nu\frac{F}{A}\frac{\nu}{1-\nu} - \nu\frac{F}{A} = -\nu\frac{F}{A}\left(\frac{\nu}{1-\nu} + 1\right)$$

$$= -\nu\frac{F}{A}\frac{\cancel{\nu} + 1 \cancel{-\nu}}{1-\nu} = -\nu\frac{F}{A}\frac{1}{1-\nu} = \underline{\sigma_x}$$

in (3)

$$\frac{\Delta h}{h} = \frac{1}{E}\left[-\frac{F}{A} - \nu\left(-\frac{F}{A}\frac{\nu}{1-\nu} - \frac{F}{A}\frac{\nu}{1-\nu}\right)\right]$$

$$= \frac{1}{E}\left[-\frac{F}{A} + 2\nu^2\frac{F}{A}\frac{1}{1-\nu}\right] = \frac{1}{E}\left[\frac{2\nu^2}{1-\nu} - 1\right]\frac{F}{A}$$

$$\Delta h = \frac{F \cdot h}{E \cdot A}\left(\frac{2\nu^2 - 1 + \nu}{1-\nu}\right) = \frac{160 \cdot 10^3\,\text{N} \cdot 60\,\text{mm}}{2.3 \cdot 10^5\,\text{N}\,\text{mm}^{-2} \cdot (40\,\text{mm})^2} \checkmark \; \frac{2 \cdot (0.3)^2 - 1 + 0.3}{1 - 0.3}$$

$$\boxed{\Delta h = -0.0194\,\text{mm}}$$

(2) $\Delta T = 100\,K$

RB: $\sigma_z = 0$, $\varepsilon_x, \varepsilon_y = 0$

aus Hooke'schem Gesetz inkl. Temp.

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \cancel{\sigma_z}^{=0}\right)\right] + \alpha_T \Delta T \overset{!}{=} 0 \qquad (1)$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \cancel{\sigma_z}^{=0}\right)\right] + \alpha_T \Delta T \overset{!}{=} 0 \qquad (2)$$

$$\varepsilon_z = \frac{1}{E}\left[\cancel{\sigma_z}^{=0} - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha_T \Delta T = \frac{\Delta h}{h} \qquad (3)$$

aus (1) $\frac{1}{E}\sigma_x = \frac{\nu}{E}\sigma_y - \alpha_T\Delta T \rightarrow \sigma_x = \nu\sigma_y - E\alpha_T\Delta T \quad (1^*)$

aus (2) $\frac{1}{E}\sigma_y = \frac{\nu}{E}\sigma_x - \alpha_T\Delta T \rightarrow \sigma_y = \nu\sigma_x - E\alpha_T\Delta T \quad (2^*)$

(2^*) in (1^*): $\sigma_x = \nu\left(\nu\sigma_x - E\alpha_T\Delta T\right) - E\alpha_T\Delta T$

$$\sigma_x(1-\nu^2) = -E\alpha_T\Delta T(1+\nu)$$

$$\underline{\sigma_x} = -E\alpha_T\Delta T\frac{\cancel{1+\nu}}{(1-\nu)\cancel{(1+\nu)}} = \underline{-E\alpha_T\Delta T\frac{1}{1-\nu}}$$

in (2^*) $\sigma_y = \nu\left(-E\alpha_T\Delta T\frac{1}{1-\nu}\right) - E\alpha_T\Delta T$

$$= -E\alpha_T\Delta T\left(\frac{\nu}{1-\nu} + 1\right) = -E\alpha_T\Delta T\frac{\cancel{\nu}+1\cancel{-\nu}}{1-\nu}$$

$$\underline{\sigma_y} = -E\alpha_T\Delta T\frac{1}{1-\nu} = \underline{\sigma_x}$$

in (3) $\Delta h = \frac{h}{\cancel{E}}\left[-\nu\left(-2\cancel{E}\alpha_T\Delta T\frac{1}{1-\nu}\right)\right] + \alpha_T\Delta T h$

$$= h\alpha_T\Delta T\left(\frac{2\nu}{1-\nu} + 1\right) = h\alpha_T\Delta T\frac{\cancel{2\nu}+1\cancel{-\nu}}{1-\nu}$$

$$\underline{\Delta h = h\alpha_T\Delta T\frac{1+\nu}{1-\nu}} = 60\,mm \cdot 1.2\cdot10^{-5}\,\cancel{K}^{-1}\,100\,\cancel{K}\,\frac{1+0.3}{1-0.3} = \underline{0.13\,mm}$$

✓

positiv ✓
(Erwärmung)