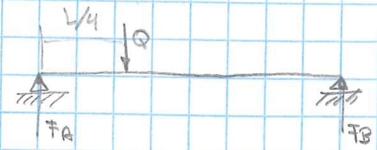
Bsp. 78:

(1) Lagerreaktionen



$$Q = q_0 \cdot \frac{L}{2}$$

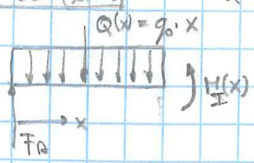
$$\sum F_y: \quad F_A - Q + F_B = 0$$

$$\sum M^{(A)}: \quad -Q \cdot L/4 + F_B \cdot L = 0 \longrightarrow F_B = \frac{Q}{4} = \underline{\frac{q_0 L}{8}}$$

$$F_A = Q - F_B = q_0\left(\frac{L}{2} - \frac{L}{8}\right) = \underline{\frac{3 q_0 L}{8}}$$

(2) Momentenverlauf

1. Feld (links) $\quad x < \frac{L}{2}$



$$\sum M_S = 0: \quad M_I(x) - F_A \cdot x + Q(x) \cdot \frac{x}{2} = 0$$

$$M_I(x) = F_A \cdot x - q_0 \frac{x^2}{2} = \frac{3 q_0 L}{8} x - q_0 \frac{x^2}{2}$$

Maximalwert: $\quad \frac{d M_I(x)}{dx} = 0: \quad \frac{3 q_0 L}{8} - q_0 x = 0$

$$\longrightarrow \underline{x = \frac{3L}{8}}$$

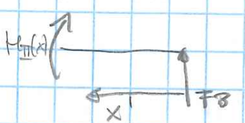
$$M_I^{max} = M_I\left(\frac{3L}{8}\right) = \frac{3 q_0 L}{8} \cdot \frac{3L}{8} - \frac{q_0}{2} \cdot \frac{9 L^2}{64}$$

$$= \left(\frac{9}{64} - \frac{9}{2 \cdot 64}\right) L^2 q_0 = \frac{18 - 9}{128} L^2 q_0$$

$$\underline{M_I^{max} = \frac{9}{128} q_0 L^2} \quad (\sim 0.07)$$

✓ größter Wert am Träger

2. Feld (rechts) $\quad x' \leq \frac{L}{2}$

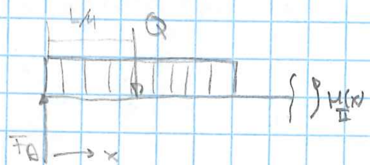


$$M_{II}(x') = F_B \cdot x'$$

$$M_{II}^{max} = M_{II}\left(x' = \frac{L}{2}\right) = q_0 \frac{L^2}{16} \quad (\sim 0.06)$$

größter möglicher Momentwert

Alternativ: 2. Feld (rechts) $\quad x \geq \frac{L}{2}$



$$\sum M_S = 0: \quad M_{II}(x) - F_A \cdot x + Q \cdot \left(x - \frac{L}{4}\right) = 0$$

$$M_{II}(x) = F_A \cdot x - q_0 \cdot \frac{L}{2}\left(x - \frac{L}{4}\right)$$

$$= \frac{3L}{8} q_0 \cdot x - \frac{L}{2} q_0 x + q_0 \frac{L^2}{8}$$

$$M_{II}(x) = q_0\left(\frac{L^2}{8} - \frac{L}{8} x\right)$$

Maximalwert: $\quad \frac{d M_{II}(x)}{dx} = 0: \quad \cancel{q_0} \frac{L}{8} = 0$

M_{max} bei $\frac{L}{2}$: (weil dort x minimal!) $\quad M_{max_{II}}\left(x = \frac{L}{2}\right) = q_0\left(\frac{L^2}{8} - \frac{L^2}{16}\right) = \underline{q_0 \frac{L^2}{16}} \quad (\sim 0.06)$

$$M_{max} = M_I^{max} = \frac{9}{128} q_0 L^2 = \frac{9}{128} \cdot 3 \cdot 3000^2 = \underline{1.898 \cdot 10^6 \text{ Nmm}}$$

maximale Biegespannung: $\quad \sigma_{max} = \frac{M_{max}}{W_y} \leq \sigma_{zul} \longrightarrow \boxed{W_y \geq \frac{M_{max}}{\sigma_{zul}}}$

$$\sigma_{zul} = \frac{\sigma_{yield}}{S} = \frac{225 \text{ Nmm}^{-2}}{1.2} = 187.5 \text{ Nmm}^{-2}$$

$$\boxed{W_y = \frac{I_y}{e}}$$

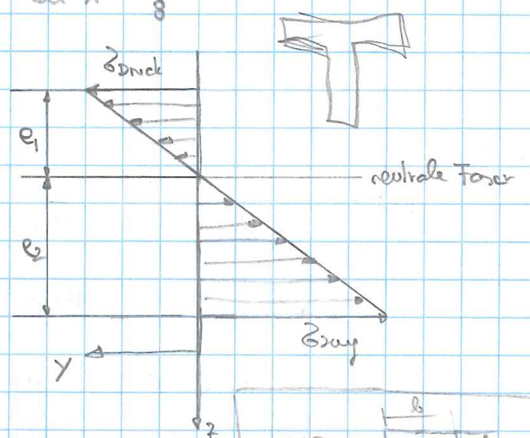
$$W_{erf} \geq \frac{1.898 \cdot 10^6\, Nmm}{187.5\, Nmm^{-2}} = 10\,122.7\, mm^3$$

$W_{erf} \geq 10.123\, cm^3 \longrightarrow$ Wähle aus Dübbel T80 mit $\underline{W_y = 12.8\, cm^3}$.

Zusatzbsp. (TEST 35):

aus vorherigem Bsp.: $M_{max} = 1.898 \cdot 10^6\, Nmm$ bei $x = \frac{3L}{8}$

(1) Lage der neutralen Faser?
(bei reiner Biegung im Schwerpunkt)



e_1 & e_2:

$$e_2 = \frac{\sum z_i \cdot A_i}{\sum A_i}$$

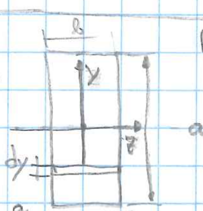
$$= \frac{\left(-\frac{a}{2}\right)\cdot a \cdot b - \left(a + \frac{b}{2}\right) a \cdot b}{2ab}$$

$$= -\frac{a}{4} - \frac{2a}{4} - \frac{b}{4}$$

$$= -\left(\frac{3a}{4} + \frac{b}{4}\right) = \underline{(-)8cm}$$

$$e_1 = -(a + b - e_2) = \underline{(-)4cm}$$

NR:



Analog: $\int z\, dA$ Flächenmoment 1. Grades (SP)

Flächenträgheitsmoment ⇒ aus Flächenträgheitsmoment

$$J_z = \int_{-a/2}^{a/2} y^2\, dA = \int_{-a/2}^{a/2} y^2 (b\, dy) = b \left.\frac{y^3}{3}\right|_{-a/2}^{a/2}$$

$$= \frac{b}{3}\left(\frac{a^3}{8} + \frac{a^3}{8}\right) = \frac{2ba^3}{24} = \underline{\frac{a^3 b}{12}}$$

$$J_y = \int_{-b/2}^{b/2} z^2\, dA = \int_{-b/2}^{b/2} z^2 (a\, dz) = a \left.\frac{z^3}{3}\right|_{-b/2}^{b/2}$$

$$= \frac{a}{3}\left(\frac{b^3}{8} + \frac{b^3}{8}\right) = \underline{\frac{ab^3}{12}}$$

$J_{yz} = -\int yz\, dA$ Deviationsmoment

Satz von Steiner:

$I_y = I_{y'} + d_y^2 A$ vom Schwerpunkt weg

$I_z = I_{z'} + d_z^2 A$

um Achse ↑ um SP

(2) Flächenträgheitsmoment

$$J_y = \underbrace{\frac{a^3 b}{12}}_{J_{1s}} + \underbrace{s_1^2 \cdot A_1}_{\text{Steiner } 1} + \underbrace{\frac{ab^3}{12}}_{J_{2s}} + s_2^2 \cdot A_2$$

$$= \frac{a^3 b}{12} + \left(e_2 - \frac{a}{2}\right)^2 \cdot a \cdot b + \frac{ab^3}{12} + \left(e_1 - \frac{b}{2}\right)^2 \cdot a \cdot b$$

$$= \frac{10^3 \cdot 2}{12} + \left(+8 - \frac{10}{2}\right)^2 \cdot 10 \cdot 2 + \frac{10 \cdot 2^3}{12} + \left(4 - \frac{2}{2}\right)^2 \cdot 10 \cdot 2$$

$$= 166.67 + 180 + 6.67 + 180$$

$$\underline{J_y = 533.34\, cm^4}$$

(3) Widerstandsmomente:

$$W_{y_1} = \frac{J_y}{e_1} = \frac{533.34\, cm^4}{4\, cm} = \underline{133.33\, cm^3}$$

$$W_{y_2} = \frac{J_y}{e_2} = \frac{533.34\, cm^4}{8\, cm} = \underline{66.67\, cm^3}$$

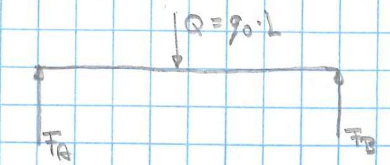
(4) Spannungen an den Randfasern

$$\sigma_{1Rand} = \frac{M_{max}}{W_{y_1}} = \frac{1.898 \cdot 10^5\, Ncm}{133.33\, cm^3} = 1423.5\, N cm^{-2} = 14.24\, Nmm^{-2} \text{ (Druck)}$$

$$\sigma_{2Rand} = \frac{M_{max}}{W_{y_2}} = \frac{1.898 \cdot 10^5\, Ncm}{66.67\, cm^3} = 2846.8\, Ncm^{-2} = 28.47\, Nmm^{-2} \text{ (Zug)}$$

Bsp. 79:

(1) Auflager

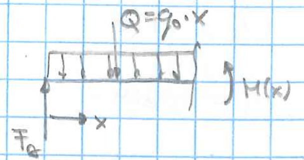


$$\sum F_y = 0: \quad F_A - q_0 \cdot L + F_B = 0$$

$$\sum M^{(A)} = 0: \quad -q_0 \cdot \frac{L^2}{2} + F_B L = 0$$

$$F_B = q_0 \frac{L}{2} \quad , \quad F_A = q_0\left(L - \frac{L}{2}\right) = q_0 \frac{L}{2}$$

(2) Schnittgrößen



$$\sum M_S = 0: \quad M(x) + q_0 \cdot \frac{x^2}{2} - F_A \cdot x = 0$$

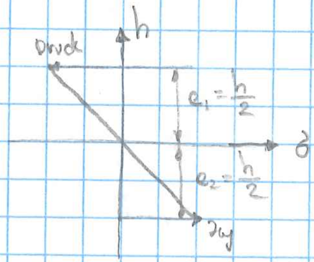
$$M(x) = q_0 \frac{L}{2} x - q_0 \frac{x^2}{2}$$

Extremwert: $\frac{dM(x)}{dx} \overset{!}{=} 0: \quad \cancel{q_0} \frac{L}{2} - \cancel{q_0} \cdot x = 0$

$$\underline{x = \frac{L}{2}}$$

$$\underline{M_{max}} = M(L/2) = q_0 \frac{L^2}{4} - q_0 \frac{1}{2} \frac{L^2}{4} = q_0 \left(\frac{L^2}{4} - \frac{L^2}{8}\right) = \underline{q_0 \frac{L^2}{8}}$$

(3) Maximale Spannung:



$$\sigma_{max} = \frac{M_{max}}{W_y} \quad , \quad W_y = \frac{I_y}{e_1} = \frac{I_y}{h/2}$$

$$I_y = \frac{b \cdot h^3}{12}$$

also gilt: $\underline{W_y} = \frac{2 b \cdot h^{\cancel{3}2}}{12 \cancel{h}} = \underline{\frac{b \cdot h^2}{6}}$

$$\underline{\sigma_{max}} = \frac{M_{max}}{W_y} = q_0 \frac{L^2}{\cancel{8}4} \frac{\cancel{6}3}{b \cdot h^2} = \underline{\frac{3 q_0 L^2}{4 b h^2}} = \frac{3 \cdot 6.4 \cdot 10^3 \, Nm^{-1} \cdot (3.5 m)^2}{4 \cdot (0.14 m) \cdot (0.24 m)^2} = \underline{7.292 \cdot 10^6 \, Nm^{-2}}$$

$\boxed{\sigma_{max}^{Rand} = 7.29 \, MPa}$ Zug unten / Druck oben

(4) Spannungsverteilung:

$$W_y(z) = \frac{I_y}{z} \quad \rightarrow \quad \sigma_{max}(z) = \frac{M_{max}}{I_y} \cdot z$$

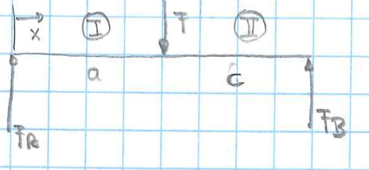
$$\Rightarrow \quad \sigma_{max}\left(-\frac{h}{2}\right) = -\sigma_{max}^{Rand} \quad \text{(Druck) oben}$$

$$\sigma_{max}\left(\frac{h}{2}\right) = \sigma_{max}^{Rand} \quad \text{(Zug) unten}$$

$$\sigma_{max}(z=0) = 0 \quad \text{neutrale Faser (Schwerpunkt!)}$$

Zusatzbsp. (FEST 32)

(1) Lagerreaktionen:



$\sum F_y = 0: \quad F_A - F + F_B = 0$

$\sum M^{(A)} = 0: \quad -F \cdot a + F_B(a+c) = 0$

$F_B = F \frac{a}{a+c} \quad , \quad F_A = F\left(1 - \frac{a}{a+c}\right) = F \frac{c}{a+c}$

(2) Schnittgrößen:

$$\left.\begin{aligned} M_I(x) &= F_A \cdot x \\ M_{II}(x) &= F_A \cdot x - F \cdot (x-a) \end{aligned}\right\}$$

Maximum bei $x = a$: & $M_{II}(x)$

$M_{max} = M_{II}(x=a) = F_A \cdot a - F(a-a)$

$= F \frac{c}{a+c} \cdot a = 3000N \cdot \frac{3m}{5m+3m} \cdot 5m$

$\underline{M_{max} = 5625 Nm}$

(3) Maximale Spg. → max. Durchmesser

$$\sigma_{max} = \frac{M_{max}}{W} = \frac{M_{max}}{J_y} \cdot z_{max} = \frac{M_{max}}{J_y} \frac{h}{2}$$

$$J_y = J_y^{rechteck} - J_y^{kreis} = \frac{b h^3}{12} - \frac{d^4 \pi}{64}$$

mit $\sigma_{max} \leq \sigma_{zul}$ folgt daher

$$\frac{M_{max}}{J_y} \cdot \frac{h}{2} \leq \sigma_{zul}$$

$$J_y \geq \frac{M_{max}}{\sigma_{zul}} \frac{h}{2}$$

$$\frac{b h^3}{12} - \frac{d^4 \pi}{64} \geq \frac{M_{max}}{\sigma_{zul}} \frac{h}{2}$$

$$d \leq \sqrt[4]{\left(\frac{b h^3}{12} - \frac{M_{max}}{\sigma_{zul}} \frac{h}{2}\right) \frac{64}{\pi}} = \sqrt[4]{\left(\frac{10 \cdot 10^3}{12} - \frac{5.625 \cdot 10^5}{5000} \frac{10}{2}\right) \frac{64}{\pi}}$$

$$\boxed{d \leq 8.61\,cm}$$

Bsp. 80:

$$I_y = \frac{h^4}{12} \longrightarrow \underline{W_y} = \frac{I_y}{z_{max}} = \frac{h^{\cancel{4}3}}{\cancel{12}_6}\cdot\frac{2}{\cancel{h}} = \frac{h^3}{6}$$ für quadratisch Querschnitt. (immer)

$z_{max} = \frac{h}{2}$ ... Randfaserabstand

mit $h = h(x)$ in der Form:

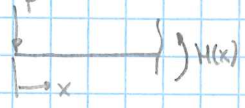
$$h(x) = h_A + \frac{h_B - h_A}{L}\cdot x \qquad \text{Geradengl.}$$

$$= h_A + \frac{3h_A - h_A}{L}\,x = h_A + \frac{2h_A}{L}\,x = h_A\left(1 + \frac{2x}{L}\right)$$

gilt dann:

$$W_y(x) = \frac{h(x)^3}{6} = \frac{1}{6}h_A^3\left(1 + \frac{2x}{L}\right)^3$$

$$\sigma(x) = \frac{|M(x)|}{W_y(x)}$$ wobei gilt



$M(x) = -P\cdot x$

$$\sigma(x) = \frac{6P\cdot x}{h_A^3\left(1+\frac{2x}{L}\right)^3}$$

Maximalwert: $$\frac{d\sigma(x)}{dx} \overset{!}{=} 0: \quad \frac{6P\left[h_A^3\left(1+\frac{2x}{L}\right)^3 - x\,h_A^3\cdot 3\cdot\left(1+\frac{2x}{L}\right)^2\frac{2}{L}\right]}{\left[h_A^3\left(1+\frac{2x}{L}\right)^3\right]^2} = 0$$

NB: $\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$ Quotientenregel

$$\cancel{h_A^3}\left(1+\frac{2x}{L}\right)^{\cancel{3}} - x\,\cancel{h_A^3}\cdot 3\cancel{\left(1+\frac{2x}{L}\right)^2}\frac{2}{L} = 0$$

$$1 + \frac{2x}{L} - 6\,\frac{x}{L} = 0$$

$$1 - \frac{4x}{L} = 0 \longrightarrow \boxed{x = \frac{L}{4}}$$ Stelle der max. Spg.

damit $$\sigma_{max} = \frac{{}^3\cancel{6}P\cdot\frac{L}{\cancel{2}4}}{h_A^3\left(1+\frac{\cancel{2}L}{\cancel{4}L_2}\right)^3} = \frac{3P\cdot L}{2h_A^3\left(1+\frac{1}{2}\right)^3} = \frac{3P\cdot L}{2h_A^3\left(\frac{3}{2}\right)^3} = \frac{3P\,L}{2h_A^3\,\frac{27}{8}} = \frac{\cancel{3}\cdot\cancel{8}^4\cdot P\cdot L}{\cancel{2}\cdot\cancel{27}_9\cdot h_A^3}$$

$$\boxed{\sigma_{max} = \frac{4PL}{9h_A^3}}$$

Spg. an der Stelle $x = L$

$$\sigma_B = \sigma(x=L) = \frac{6P\cdot L}{h_A^3(1+2)^3} = \frac{6PL}{27h_A^3}$$

dann ist

$$\frac{\sigma_{max}}{\sigma_B} = \frac{4\cancel{PL}\cdot 27\cancel{h_A^3}}{9\cancel{h_A^3}\cdot 6\cancel{PL}} = \frac{108}{54} = \underline{2}$$

Allg.: schiefe Biegung

(Gerade: $\sigma = \frac{M c_{max}}{I} = \frac{M}{W}$)

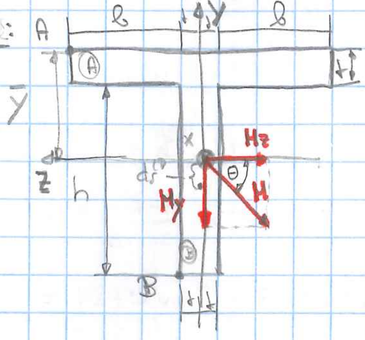
(a) nicht symmetrischer Trägerquerschnitt, aber Biegemomente um Hauptachsen (von $\underline{\underline{I}}$) betrachtet

(b) Beliebig wirkendes Moment
→ in Komponenten zu Hauptachsen aufspalten & Komponentenweise die Spgen. ausrechnen.

Wichtig ist immer der Richtungssinn des KS (rechtshändig): $\boxed{\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}}$ Normalspg.

! reine Biegung, sonst noch Normalkraft $\frac{F}{A}$

Bsp. 81:



Momentenkomp.: $M_y = -M \cdot \sin\theta = -15 \cdot \sin(30°) = -7.5\,kNm = -7.5 \cdot 10^6\,Nmm$

$M_z = -M \cos\theta = -15 \cdot \cos(30°) = -12.99\,kNm = -12.99 \cdot 10^6\,Nmm$

x aus Blatt heraus

Schwerpunkt: $\bar{z} = 0$

$$\bar{y} = \frac{\sum_i \bar{y}_i A_i}{\sum A_i} = \frac{\frac{d}{2} \cdot d \cdot (2b+2d) + (d+\frac{h}{2})\, 2t \cdot h}{d(2b+2d) + 2t\,h}$$

$$\bar{y} = 4.83\,cm = 48.3\,mm$$

Flächenträgheitsmomente:

$$I_z = I_{Sz}^{(A)} + A^{(A)} \underbrace{(\bar{y} - \tfrac{d}{2})}_{d_S^{(A)}}{}^2 + I_{Sz}^{(B)} + A^{(B)} \cdot \underbrace{(d + \tfrac{h}{2} - \bar{y})}_{d_S^{(B)}}{}^2 = \frac{(2b+2t) \cdot d^3}{12} + (2b+2t) \cdot d \cdot (\bar{y} - \tfrac{d}{2})^2 + \frac{h^3 \cdot 2d}{12}$$
$$+ h\,2t \cdot (d + \tfrac{h}{2} - \bar{y})^2$$

$$I_z = 1 + 224.99 + 288 + 113.014 = \underline{627\,cm^4} = \underline{627 \cdot 10^4\,mm^4}$$

$$I_y = I_{Sy}^{(A)} + I_{Sy}^{(B)} = \frac{(2b+2d)^3 d}{12} + \frac{h(2t)^3}{12} = 144 + 8 = \underline{152\,cm^4} = 152 \cdot 10^4\,mm^4$$

Biegeformel:

$$\sigma = -\frac{M_z \cdot y}{I_z} + \frac{M_y z}{I_y}$$

Punkt A: $y_A = \bar{y}$, $z_A = b + d$

$$\sigma_A = -\frac{M_z \bar{y}}{\underbrace{I_z}_{W_z^{(A)}}} + \frac{M_y (b+d)}{\underbrace{I_y}_{W_y^{(A)}}} = -\frac{-12.99 \cdot 10^6\,Nmm \cdot 48.3\,mm}{627 \cdot 10^4\,mm} + \frac{-7.5 \cdot 10^6\,Nmm \cdot (50+10)\,mm}{152 \cdot 10^4\,mm}$$

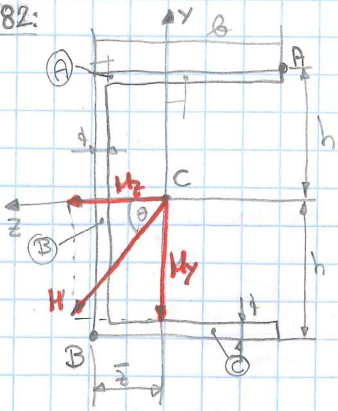
$$\underline{\sigma_A = -196\,MPa}$$

Punkt B: $y_B = -(h + d - \bar{y})$, $z_B = d$

$$\sigma_B = -\frac{M_z(-h-d+\bar{y})}{I_z} + \frac{M_y \cdot d}{I_y} = -\frac{-12.99 \cdot 10^6 \cdot (-81.7)}{627 \cdot 10^4} + \frac{-7.5 \cdot 10^6 \cdot 10}{152 \cdot 10^4}$$

$$\underline{\sigma_B = -219\,MPa}$$

Bsp. 82:



Momentenkomponenten:

$$M_y = -M \cdot \sin\theta = -692.82\,Nm = -692.82 \cdot 10^3\,Nmm$$

$$M_z = M \cdot \cos\theta = 400\,Nm = 400 \cdot 10^3\,Nmm$$

Schwerpunkt:

$$\bar{z} = \frac{\sum_i \bar{z}_i A_i}{\sum A_i} = \frac{d/2 \cdot d \cdot 2h + 2\left(\frac{b}{2}(b-d) \cdot d\right)}{2dh + 2(b-d) \cdot t} = \underline{34.17\,mm}$$

(Folie in Lsg.)

Flächenträgheitsmomente:

$$I_z = I_{Sz}^{(A)} + A^{(A)} \cdot \left(h - \frac{d}{2}\right)^2 + I_{Sz}^{(B)} + I_{Sz}^{(C)} + A^{(C)} \cdot \left(-h + \frac{d}{2}\right)^2$$

$$= \frac{d^3 \cdot (b-d)}{12} + (b-d)d\left(h - \frac{d}{2}\right)^2 + \frac{(2h)^3 d}{12} + \frac{d^3(b-d)}{12} + (b-d)d\left(-h + \frac{d}{2}\right)^2$$

$$= 19872\,mm^4 + 62325216\,mm^4 + 6.4 \cdot 10^7\,mm^4 + 1.9872 \cdot 10^4\,mm^4 + 6.2325216 \cdot 10^7\,mm^4$$

$$\underline{I_z = 1.8869 \cdot 10^8\,mm^4}$$

$$I_y = J_{Sy}^{(A)} + A^{(A)} \cdot \left(\frac{b-d}{2} + d - \bar{z}\right)^2 + J_{Sy}^{(B)} + A^{(B)} \cdot \left(\bar{z} - \frac{d}{2}\right)^2 + J_{Sy}^{(C)} + A^{(C)} \cdot \left(\frac{b-d}{2} + d - \bar{z}\right)^2$$

$$= 2\frac{d(b-d)^3}{12} + 2(b-d) \cdot d\left(\frac{b-d}{2} + d - \bar{z}\right)^2 + \frac{2hd^3}{12} + 2dh \cdot \left(\bar{z} - \frac{d}{2}\right)^2$$

$$= 5.256144 \cdot 10^6 + 7.2634 \cdot 10^6 + 5.76 \cdot 10^4 + 3.809 \cdot 10^6$$

$$\underline{I_y = 1.6386 \cdot 10^7\,mm^4}$$

Biegespannung in A & B:

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

Punkt A: $y_A = h$, $z_A = -(b - \bar{z}) = -b + \bar{z}$

$$\sigma_A = -\frac{M_z \cdot h}{I_z} + \frac{M_y(-b+\bar{z})}{I_y} = -0.424 + 4.897 = \underline{4.47\,MPa}$$

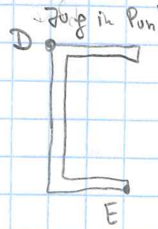
Punkt B: $y_B = -h$, $z_B = \bar{z}$

$$\sigma_B = -\frac{M_z \cdot (-h)}{I_z} + \frac{M_y \bar{z}}{I_y} = 0.424 - 1.445 = \underline{-1.02\,MPa}$$

Größte Spannungen:

Druck in Punkt D (links oben):

Zug in Punkt E (rechts unten):

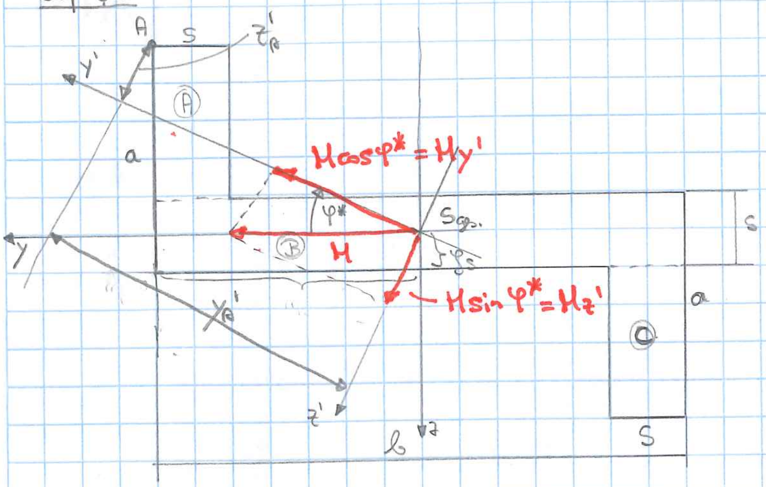


$$\sigma_D = -\frac{M_z \cdot h}{I_z} + \frac{M_y \cdot \bar{z}}{I_y} = -0.424 - 1.445 = \underline{-1.869\,MPa}$$

$$\sigma_E = -\frac{M_z \cdot (-h)}{I_z} + \frac{M_y(-b+\bar{z})}{I_y} = 0.424 + 4.897 = \underline{5.32\,MPa}$$

suche größte Kombi von oben! (σ_A, σ_B)

Bsp. 83:



Flächenträgheitsmomente bzgl. y, z, yz:

Rechteck: $I = \frac{1}{12} b h^3$

Addition & Satz von Steiner:

$$I_y = I_{sy}^{(A)} + A^{(A)} \cdot \left(\frac{a-s}{2} + \frac{s}{2}\right)^2 + I_{sy}^{(B)} + I_{sy}^{(C)} + A^{(C)} \cdot \left(\underbrace{\frac{a-s}{2} + \frac{s}{2}}_{\left(\frac{a}{2}\right)^2}\right)^2 \quad (A \,\&\, C \text{ gleich})$$

$$= 2 \frac{1}{12} s (a-s)^3 + 2(a-s) \cdot s \cdot \left(\frac{a}{2}\right)^2 + \frac{1}{12} s^3 b$$

$$I_y = 2.8125 \cdot 10^7 + 1.5 \cdot 10^8 + 3.125 \cdot 10^6 = \underline{1.8125 \cdot 10^8 \, mm^4} \checkmark$$

$$I_z = I_{sz}^{(A)} + A^{(A)} \cdot \left(\frac{b-s}{2}\right)^2 + I_{sz}^{(B)} + I_{sz}^{(C)} + A^{(C)} \cdot \left(\frac{b-s}{2}\right)^2$$

$$= 2 \frac{1}{12} s^3 (a-s) + 2 \cdot (a-s) \cdot s \cdot \left(\frac{b-s}{2}\right)^2 + \frac{1}{12} s b^3$$

$$I_z = 3.125 \cdot 10^6 + 2.344 \cdot 10^8 + 1.125 \cdot 10^8 = \underline{3.5 \cdot 10^8 \, mm^4} \checkmark$$

$$I_{yz} = I_{syz}^{(A)} + A^{(A)} \cdot \left(\frac{a}{2}\right) \cdot \left(-\frac{b-s}{2}\right) + I_{syz}^{(B)} + I_{syz}^{(C)} + A^{(C)} \left(-\frac{a}{2}\right)\left(\frac{b-s}{2}\right)$$

$$I_{yz} = 0 - 2 \cdot (a-s) \cdot s \left(\frac{a}{2}\right)\left(\frac{b-s}{2}\right) + 0 = -1.875 \cdot 10^8 \, mm^4 \checkmark$$

Hauptflächenträgheitsmomente mit Mohr'schem Trägheitskreis

$$I_{1,2} = \underbrace{\frac{I_y + I_z}{2}}_{\text{Mittelpunkt}} \pm \underbrace{\sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{xy}^2}}_{\text{Radius}}$$

$$= 2.6563 \cdot 10^8 \pm 2.0561 \cdot 10^8$$

$$I_1 = 4.712 \cdot 10^8 \, mm^4 = I_{z'}$$

$$I_2 = 6.002 \cdot 10^7 \, mm^4 = I_{y'}$$

Drehwinkel: $\tan 2\varphi^* = \frac{2 I_{yz}}{I_y - I_z} = 2.2$

$\rightarrow \underline{\varphi^* = 32.9°}$

⚠ KS umgedreht ! daher ⊖ weg!

Biegespannung in A

$$\sigma_A = -\frac{M_{z'} y_A'}{I_{z'}} + \frac{M_{y'} z_A'}{I_{y'}}$$

mit $M_{z'} = M \sin\varphi^* = 135.79\,Nm$, $M_{y'} = M\cos\varphi^* = 209.9\,Nm$, $I_{y'} = 6.002\cdot 10^{-5}\,m^4$

$y_A' = \frac{b}{2}\cos\varphi^* + \left(a-\frac{c}{2}\right)\sin\varphi^*$, $z_A' = \frac{b}{2}\sin\varphi^* - \left(a-\frac{c}{2}\right)\cos\varphi^*$, $I_{z'} = 4.712\cdot 10^{-4}\,m^4$

$y_A' = 221\,mm = 0.221\,m$ $\quad z_A' = -65.46\,mm = -0.0655\,m$

also: $\sigma_A = -\frac{135.79\,N\cancel{m}\cdot 0.221\,\cancel{m}}{4.712\cdot 10^{-4}\,m^{\cancel{4}2}} + \frac{209.9\,N\cancel{m}\cdot(-0.0655\,\cancel{m})}{6.002\cdot 10^{-5}\,m^{\cancel{4}2}}$

$\underline{\underline{\sigma_A}} = -2.928\,Nm^{-2} = \underline{\underline{-293\,kPa}}$