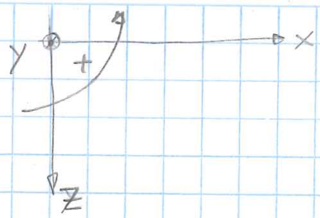
Bsp. 84:

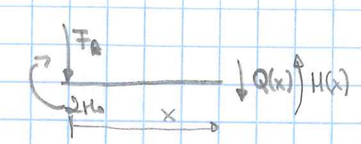
Auflager:



$$\overset{+}{\circlearrowleft}\ \sum M_i^{(B)} = 0: \ -2M_0 - M_0 + F_A \cdot L = 0$$

$$\underline{F_A = \frac{3M_0}{L}}$$

Querkraft im Balken:



$$\underline{Q(x) = -F_A = -\frac{3M_0}{L}}\ , \quad \text{NB: } M(x) = 2M_0 - F_A x = 2M_0 - 3M_0 \frac{x}{L} \qquad (0)$$

für später

DGL d. Biegelinie mit Querkraft:

$$EI w'''(x) = -Q(x) = \frac{3M_0}{L}$$

$$EI w''(x) = \frac{3M_0}{L} x + c_1 \qquad (1)$$

$$EI w'(x) = \frac{3M_0}{L}\frac{x^2}{2} + c_1 x + c_2 \qquad (2)$$

$$EI w(x) = \frac{3M_0}{L}\frac{x^3}{6} + c_1 \frac{x^2}{2} + c_2 x + c_3 \qquad (3)$$

Konstanten bestimmen, RB:

① Keine Durchbiegung in Lagern: $w(x=0) = 0$, $w(x=L) = 0$

② Biegemoment links: $M(x=0) = 2M_0 \rightarrow EI w''(x=0) = -2M_0$

Rechtsmoment pos. Gl. (0)

aus (3) mit ①:

$$EI w(x=0) = \boxed{c_3 = 0}\ , \quad EI w(x=L) = \frac{\cancel{3}M_0}{\cancel{L}}\frac{L^{\cancel{3}2}}{\cancel{6}2} + c_1 \frac{L^2}{2} + c_2 \cancel{L} = 0$$

$$\rightarrow c_2 = \frac{-M_0 L}{2} - c_1 \frac{L}{2}$$

aus (1) mit ②:

$$EI w''(x=0) = \boxed{c_1 = -2M_0} \quad \text{also} \quad \boxed{c_2 = -\frac{M_0 L}{2} + 2M_0 \frac{L}{2} = +\frac{M_0 L}{2}}$$

$$EI w(x) = +\frac{\cancel{3}M_0}{L}\frac{x^3}{\cancel{6}2} - \cancel{2}M_0 \frac{x^2}{\cancel{2}} + \frac{M_0 L}{2} x \rightarrow \left\| EI w(x) = \frac{M_0}{2L} x^3 - M_0 x^2 + \frac{M_0 L}{2} x \right\|$$

größte Durchbiegung:

setze $w' = 0$ (Gl. (2)):
$$\frac{3\cancel{M_0}}{2L} x^2 - 2\cancel{M_0} x + \frac{\cancel{M_0} L}{2} = 0 \rightarrow \frac{3}{2L} x^2 - 2x + \frac{L}{2} = 0$$
Löse quadr. Gl.

$$x_{1,2} = \frac{+2 \pm \sqrt{4 - 4 \cdot \left(+\frac{3}{2L}\right)\cdot\left(\frac{L}{2}\right)}}{+2 \cdot \frac{3}{2L}} = -\frac{2}{3}L \pm \sqrt{4-3}\ \frac{L}{3} = +\frac{2}{3}L \pm \frac{L}{3}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\cancel{x_1 = L}$, $x_2 = \frac{L}{3}$

im Lager kein Biegemoment

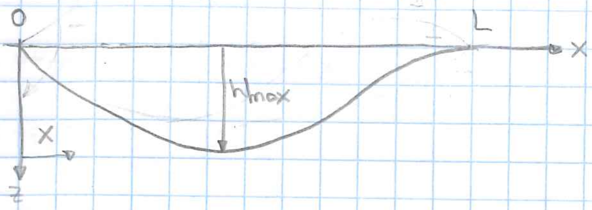
$$\rightarrow x_{max} = \frac{L}{3}\ , \quad w(x_{max}) = \frac{1}{EI}\left[+\frac{M_0}{2\cancel{L}}\frac{L^{\cancel{3}2}}{27} - M_0 \frac{L^2}{9} + \frac{M_0 L^2}{6}\right]$$

$$w(x_{max}) = \frac{M_0 L^2}{EI}\left[+\frac{1}{54} - \frac{1}{9} + \frac{1}{6}\right] = \frac{4M_0 L^2}{54 EI}\ , \quad \boxed{w(x_{max}) = \frac{2M_0 L^2}{27 EI}}$$

Steigung in den Lagern:

$$\underline{w'(x=0)} = \frac{1}{+EI} C_2 = \underline{\frac{M_0 L}{2EI}}$$

$$\underline{w'(x=L)} = \frac{1}{+EI}\left[+\frac{3M_0}{\cancel{L}}\frac{L^{\cancel{2}}}{2} - 2M_0L + \frac{M_0L}{2}\right] = \frac{M_0L}{EI}\left[\underbrace{+\frac{3}{2}-\frac{4}{2}+\frac{1}{2}}_{0}\right] = \underline{\underline{0}}$$



Bsp. 85:

DGL der Biegelinie für Streckenlast $q(x)$, wobei $q(x) = Kx + d = \frac{q_0}{l} \cdot x$

$$EIw''''(x) = +q(x)$$

$$EIw''''(x) = +\frac{q_0}{l} x$$

$$EIw'''(x) = +\frac{q_0}{l} \frac{x^2}{2} + c_1$$

$$EIw''(x) = +\frac{q_0}{l} \frac{x^3}{6} + c_1 \cdot x + c_2 \qquad (1)$$

$$EIw'(x) = +\frac{q_0}{l} \frac{x^4}{24} + c_1 \frac{x^2}{2} + c_2 x + c_3 \qquad (2)$$

$$EIw(x) = +\frac{q_0}{l} \frac{x^5}{120} + c_1 \frac{x^3}{6} + c_2 \frac{x^2}{2} + c_3 x + c_4 \qquad (3)$$

Bestimme die Konstanten, RB:

① Durchbiegung in Lagern: $w(x=0) = 0$, $w(x=l) = 0$

② Kein Biegemoment in Lagern: $w''(x=0) = 0$, $w''(x=l) = 0$

① in (3): $EIw(0) = \boxed{c_4 = 0}$

$$EIw(l) = +\frac{q_0}{\cancel{l}} \frac{l^{\cancel{5}4}}{120} + c_1 \frac{l^3}{6} + \underbrace{\cancel{c_2}}_{=0} \frac{l^2}{2} + c_3 l + \underbrace{\cancel{c_4}}_{=0} = 0$$

② in (1): $EIw''(0) = \boxed{c_2 = 0}$

$$EIw''(l) = +\frac{q_0}{l} \frac{l^3}{6} + c_1 \cdot l + \underbrace{\cancel{c_2}}_{=0} = 0$$

also: $+\frac{q_0 l^{\cancel{4}3}}{120} + c_1 \frac{l^{\cancel{3}2}}{6} + c_3 \cancel{l} = 0 \rightarrow c_3 = -\frac{q_0 l^3}{120} - c_1 \frac{l^2}{6}$

$$+\frac{q_0 l^2}{6} + c_1 \cancel{l} = 0 \rightarrow \boxed{c_1 = -\frac{q_0 l}{6}}$$

$$c_3 = -\frac{q_0 l^3}{120} + \frac{q_0 l}{6} \frac{l^2}{6} = -\frac{q_0 l^3}{120} + \frac{q_0 l^3}{36} = +\frac{q_0 l^3}{4320}(-36 + 120) = \frac{+84\, q_0 l^3}{4320}$$

$$\boxed{c_3 = \frac{+7 q_0 l^3}{360}}$$

also ist DGL d. Biegelinie:

$$\boxed{EIw(x) = +\frac{q_0}{120 l} x^5 - \frac{q_0 l}{36} x^3 + \frac{7 q_0 l^3}{360} x}$$

größte Durchbiegung: $w'(x) = 0$ (Neigung null)

$$\rightarrow +\frac{q_0}{24 l} x^4 - \frac{q_0 l}{12} x^2 + \frac{7 q_0 l^3}{360} = 0$$

$$\rightarrow \frac{q_0}{360 l}\left(+15 x^4 - 30 l^2 x^2 + 7 l^4\right) = 0$$

$$+15x^4 - 30\ell^2 x^2 + 7\ell^4 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitution: $x^2 = u \rightarrow +15u^2 - 30\ell^2 u + 7\ell^4 = 0$

$$u_{1,2} = \frac{+30\ell^2 \pm \sqrt{900\ell^4 - 420\ell^4}}{30}$$

$$u_{1,2} = \frac{+30\ell^2 \pm \sqrt{480}\,\ell^2}{30} = \frac{+30\ell^2 \pm 4\sqrt{30}\,\ell^2}{30}$$

$$u_1 = \ell^2 + 0.73\ell^2 = 1.73\ell^2$$
$$u_2 = \ell^2 - 0.73\ell^2 = 0.27\ell^2$$

$$x^2 = u \rightarrow \left.\begin{array}{l} x_{1,2} = \sqrt{u_1} = \pm 1.315\ell \\ x_{3,4} = \sqrt{u_2} = \pm 0.52\ell \end{array}\right\} \quad x_{max} = \left\{\begin{array}{l} \cancel{1.315\ell} \\ 0.52\ell \end{array}\right.$$

weil außerhalb Träger

$\underline{x_{max} = 0.52\ell}$ Ort der max. Durchbiegung

also ist max. Durchbiegung:

$$EIw(x_{max}) = +\frac{q_0}{120\ell}(0.52\ell)^5 - \frac{q_0\ell}{36}(0.52\ell)^3 + \frac{7q_0\ell^3}{360}\,0.52\ell$$

$$= \frac{q_0}{120\ell}\,0.038\ell^{\cancel{5}4} - \frac{q_0\ell^4}{36}\,0.1406 + \frac{q_0\ell^4}{360}\,3.64$$

$$w(x_{max}) = q_0\ell^4\left(3.17\cdot10^{-4} - 3.91\cdot10^{-3} + 1.01\cdot10^{-2}\right)\frac{1}{EI}$$

$$\boxed{w(x_{max}) = 6.51\cdot10^{-3}\, q_0\ell^4/EI}$$ Betrag d. max. Durchbiegung nach unten.

Momentenmaximum:

$$w'''(x) = 0: \quad +\frac{q_0}{\ell}\frac{x^2}{2} - \frac{q_0\ell}{6} = 0$$

$$\frac{x^2}{2\ell} = \frac{\ell}{6} \rightarrow x^2 = \frac{\ell^2}{3}, \quad \underline{x^M_{max} = \frac{\ell}{\sqrt{3}}}$$ Ort d. max. Moments

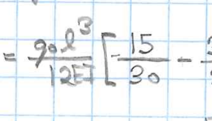
$$M_{max} = EIw''(x^M_{max}) = \frac{q_0}{6\ell}\frac{\ell^{\cancel{3}2}}{3^{3/2}} - \frac{q_0\ell}{6}\frac{\ell}{\sqrt{3}}$$

$$= \frac{q_0\ell^2}{6}(0.192 - 0.577)$$

$$\boxed{M_{max} = +0.064\, q_0\ell^2/EI}$$ max. Moment

Steigung d. Biegelinie in Lagern

links: $w'(x=0) = \frac{c_3}{EI} = \underline{\frac{7q_0\ell^3}{360EI}}$ pos. Neigung ✓



rechts: $$w'(x=\ell) = -\frac{q_0\ell^3}{24EI} - \frac{q_0\ell\cdot\ell^2}{6\cdot 2EI} + \frac{7q_0\ell^3}{360EI} = \frac{q_0\ell^3}{12EI}\left[\frac{1}{2} - 1 + \frac{7}{30}\right] = \frac{q_0\ell^3}{12EI}\left[\frac{15}{30} - \frac{30}{30} + \frac{7}{30}\right]$$

$$= -\frac{q_0\ell^3}{12EI}\frac{8}{30} = -\frac{2q_0\ell^3}{90EI} = \underline{-\frac{q_0\ell^3}{45EI}}$$ neg. Neigung ✓

Bsp. 86:

$$EJw''''(x) = +q(x) = +q_0$$ Konstante Streckenlast

$$EJw'''(x) = +q_0 x + c_1$$

$$EJw''(x) = +q_0 \frac{x^2}{2} + c_1 x + c_2$$

$$EJw'(x) = +q_0 \frac{x^3}{6} + c_1 \frac{x^2}{2} + c_2 x + c_3$$

$$EJw(x) = +q_0 \frac{x^4}{24} + c_1 \frac{x^3}{6} + c_2 \frac{x^2}{2} + c_3 x + c_4$$

RB, Einspannungen: $w = 0,\ w' = 0$

d.h. $w(0) = w(l) = 0$
$w'(0) = w'(l) = 0$

$w(0) = \boxed{c_4 = 0}$

$w'(0) = \boxed{c_3 = 0}$

$$w(l) = +q_0 \frac{l^4}{24} + c_1 \frac{l^3}{6} + c_2 \frac{l^2}{2} = 0$$

$$w'(l) = +q_0 \frac{l^3}{6} + c_1 \frac{l^2}{2} + c_2 l = 0 \rightarrow c_2 = -q_0 \frac{l^2}{6} - c_1 \frac{l}{2}$$

in $w(l)$: $+q_0 \frac{l^4}{24} + c_1 \frac{l^3}{6} - q_0 \frac{l^2}{6} \frac{l^2}{2} - c_1 \frac{l}{2} \frac{l^2}{2} = 0$

$$c_1 \left(\frac{l^3}{6} - \frac{l^3}{4} \right) = q_0 l^4 \left(-\frac{1}{12} + \frac{1}{24} \right)$$

$$c_1 \left(\frac{4l^3 - 6l^3}{24} \right) = q_0 l^4 \frac{-2+1}{24}$$

$$c_1 = \frac{-q_0 l^4}{-2l^3} \rightarrow \boxed{c_1 = -q_0 \frac{l}{2}}$$

$$c_2 = -q_0 \frac{l^2}{6} + q_0 \frac{l^2}{4} = q_0 \left(\frac{-4l^2 + 6l^2}{24} \right)$$

$$\boxed{c_2 = \frac{q_0 l^2}{12}}$$

$$EJw(x) = q_0 \frac{x^4}{24} - q_0 \frac{l}{2} \frac{x^3}{6} + q_0 \frac{l^2}{12} \frac{x^2}{2}$$

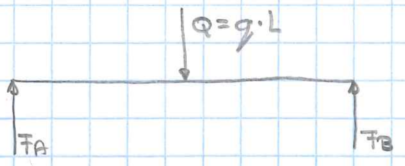
$$\| \ EJw(x) = \frac{q_0}{24} \left(x^4 - 2l x^3 + l^2 x^2 \right) \ \|$$

$$EJw''(x) = q_0 \frac{x^2}{2} - q_0 \frac{l}{2} x + q_0 \frac{l^2}{12}$$

$$\| \ EJw''(x) = \frac{q_0}{2} \left(x^2 - l x + \frac{l^2}{6} \right) \ \|$$

Bsp. 87.

Auflager:



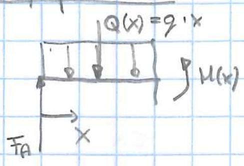
$$\Sigma F_{y_i} = 0: \quad F_A + F_B - Q = 0$$

$$\Sigma M_i^{(A)} = 0: \quad q \cdot L \cdot \frac{L}{2} - F_B \cdot L = 0$$

$$\underline{F_B = q\frac{L}{2}}$$

$$\underline{F_A} = q \cdot L - q\frac{L}{2} = \underline{q\frac{L}{2}}$$

Schnittgröße Moment:



$$M(x) = F_A \cdot x - q \cdot \frac{x^2}{2} = qL\frac{x}{2} - q\frac{x^2}{2}$$

DGL der Biegelinie:

$$EJw''(x) = -M(x) = -\frac{qL}{2}x + \frac{q}{2}x^2$$

$$EJw'(x) = -\frac{qL}{2}\frac{x^2}{2} + \frac{q}{2}\frac{x^3}{3} + c_1 = -\frac{qL}{4}x^2 + \frac{q}{6}x^3 + c_1$$

$$EJw(x) = -\frac{qL}{4}\frac{x^3}{3} + \frac{q}{6}\frac{x^4}{4} + c_1 \cdot x + c_2 = -\frac{qL}{12}x^3 + \frac{q}{24}x^4 + c_1 x + c_2$$

RB: $w(0) = 0 \longrightarrow \boxed{c_2 = 0}$

$w(x=L) = \Delta x$ von Feder, also: $F_B = k\Delta x \rightarrow q\frac{L}{2} = k\Delta x = \frac{48EJ}{L^3}\Delta x$

$$\Delta x = \frac{qL^4}{96EJ}$$

also $w(L) = \frac{qL^4}{96EJ}$

aus DGL: $w(L) = -\frac{qL^{\cancel{4}3}}{12} + \frac{qL^{\cancel{4}3}}{24} + c_1\cancel{L} = \underbrace{\frac{qL^{\cancel{4}3}}{96EJ}}_{=\Delta x}(\cdot EJ)$

$$c_1 = \frac{qL^3}{96} + \frac{2qL^3 - qL^3}{24} = \frac{qL^3}{96}(1+4) \rightarrow \boxed{c_1 = +\frac{5qL^3}{96}}$$

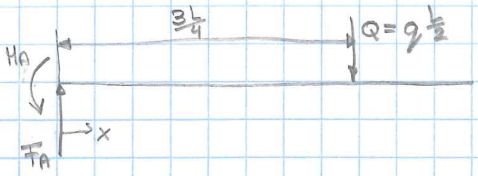
$$EJw(x) = -\frac{qL}{12}x^3 + \frac{q}{24}x^4 + \frac{5qL^3}{96}x \quad \boxed{w(x) = \frac{q}{96EJ}\left[4x^4 - 8Lx^3 + 5L^3x\right]}$$

Neigung in A:

$w'(x=0) \Rightarrow \quad +EJw'(0) = c_1 = +\frac{3qL^3}{96} \rightarrow \boxed{w'(0) = +\frac{3qL^3}{96EJ}} = \theta_A$ positiv ✓

Bsp. 88:

Auflager:



$\underline{F_A} = Q = \underline{q\frac{L}{2}}$

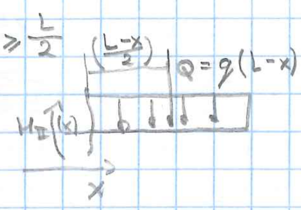
$\underline{M_A} = Q \cdot \frac{3L}{4} = \underline{q\frac{3L^2}{8}}$

Schnittgrößen:

Feld I: $x < \frac{L}{2}$

$M_I(x) = F_A \cdot x - M_A = q\frac{L}{2} \cdot x - q\frac{3L^2}{8}$

Feld II: $x \geq \frac{L}{2}$



$M_{II}(x) = -Q \cdot \frac{L-x}{2} = -q\frac{(L-x)^2}{2} = -q\frac{L^2}{2} - q\frac{x^2}{2} + \cancel{2}q\frac{Lx}{\cancel{2}}$

DGL-Biegelinie Feld I:

$EI\,w_I''(x) = -M_I(x) = -q\frac{L}{2}x + q\frac{3L^2}{8}$

$EI\,w_I'(x) = -q\frac{L}{2}\frac{x^2}{2} + q\frac{3L^2}{8}x + c_1$

$EI\,w_I(x) = -q\frac{L}{12}x^3 + q\frac{3L^2}{16}x^2 + c_1 x + c_2$

DGL-Biegelinie Feld II:

$EI\,w_{II}''(x) = -M_{II}(x) = +q\frac{L^2}{2} + q\frac{x^2}{2} - qLx$

$EI\,w_{II}'(x) = +q\frac{x^3}{6} - qL\frac{x^2}{2} + q\frac{L^2}{2}x + c_3$

$EI\,w_{II}(x) = +q\frac{x^4}{24} - qL\frac{x^3}{6} + qL^2\frac{x^2}{4} + c_3 x + c_4$

RB: $w_I(0) = 0$
$w_I'(0) = 0$

Stetigkeit: $w_I\left(\frac{L}{2}\right) = w_{II}\left(\frac{L}{2}\right)$!
$w_I'\left(\frac{L}{2}\right) = w_{II}'\left(\frac{L}{2}\right)$

$w_I(0) = \boxed{c_2 = 0}$

$w_I'(0) = \boxed{c_1 = 0}$

$w_I'\left(\frac{L}{2}\right) = -q\frac{L}{4}\frac{L^2}{4} + q\frac{3L^2}{8}\frac{L}{2} = +q\frac{1}{6}\frac{L^3}{8} - q\frac{L}{2}\frac{L^2}{4} + q\frac{L^2}{2}\frac{L}{2} + c_3 = w_{II}'\left(\frac{L}{2}\right)$

$-q\frac{L^3}{16} + q\frac{3L^3}{16} = +q\frac{L^3}{48} - q\frac{L^3}{8} + q\frac{L^3}{4} + c_3$

$c_3 = qL^3\left(-\frac{1}{48} + \frac{6}{48} - \frac{12}{48}\right) + qL^3\underbrace{\frac{2}{16}}_{\frac{6}{48}}$

$$\boxed{c_3 = -\frac{qL^3}{48}}$$

$$w_I\left(\frac{L}{2}\right) = -q\frac{L}{12}\frac{L^3}{8} + q\frac{3L^2}{16}\frac{L^2}{4} = +q\frac{L^4}{384} - qL\frac{L^3}{48} + q\frac{L^2}{4}\frac{L^2}{4} - \frac{qL^3}{48}\frac{L}{2} + c_4 = w_{II}\left(\frac{L}{2}\right)$$

$$c_4 = qL^4\left(\cancel{-\frac{1}{96}} + \underset{(6)}{\frac{3}{64}} - \frac{1}{384} + \underset{(8)}{\frac{1}{48}} - \underset{(24)}{\frac{1}{16}} + \cancel{\frac{1}{96}}\right)$$

$$c_4 = qL^4\left(+\frac{18}{384} - \frac{1}{384} + \frac{8}{384} - \frac{24}{384}\right)$$

$$\boxed{c_4 = \frac{1}{384}qL^4}$$

Also gilt:

$$EI\,w_I(x) = -\frac{qL}{12}x^3 + \frac{3qL^2}{16}x^2 \qquad w_I = \frac{q}{48EI}\left(9L^2x^2 - 4Lx^3\right)$$

$$EI\,w_{II}(x) = +\frac{q}{24}x^4 - \frac{qL}{6}x^3 + \frac{qL^2}{4}x^2 - \frac{qL^3}{48}x + \frac{qL^4}{384}$$

$$w_{II} = \frac{q}{384EI}\left(16x^4 - 64Lx^3 + 96L^2x^2 - 8L^3x + L^4\right)$$