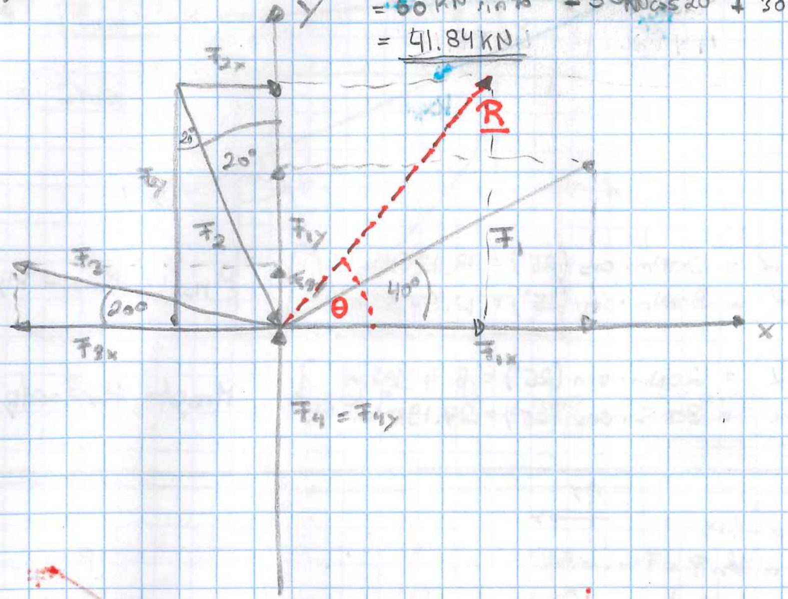
Bsp 1:

$$R_x = \sum_i F_{ix}$$
$$= F_{1x} + F_{2x} + F_{3x} = 60\,kN \cos 40^\circ + 50\,kN \sin 20^\circ - 30\,kN \cos 20^\circ = \underline{34{,}87\,kN}$$

$$R_y = \sum_i F_{iy} = F_{1y} + F_{2y} + F_{3y} + F_{4y} = F_1 \sin 40^\circ - F_2 \cos 20^\circ + F_3 \sin 20^\circ + F_4 =$$
$$= 60\,kN \sin 40^\circ - 50\,kN \cos 20^\circ + 30\,kN \sin 20^\circ + 40\,kN$$
$$= \underline{41.84\,kN}$$



$$\underline{R} = R_x \underline{e_x} + R_y \underline{e_y} = \underline{(34.87\,\underline{e_x} + 41.84\,\underline{e_y})\,kN}$$
$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(34.87\,kN)^2 + (41.84\,kN)^2} = \underline{54.47\,kN}$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{41.84\,kN}{34.87\,kN} = 1.2 \rightarrow \theta = \tan^{-1}(1.2) = \underline{50.2^\circ}$$

Bsp 2:

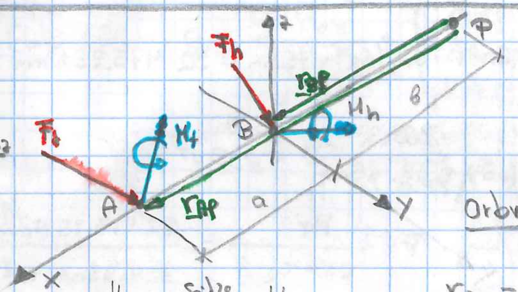
$\underline{F}_t = -50N\,\underline{e_x} + 80N\,\underline{e_y} - 158N\,\underline{e_z}$

$\underline{F}_h = -20N\,\underline{e_x} + 60N\,\underline{e_y} - 250N\,\underline{e_z}$

$\underline{M}_t = -6Nm\,\underline{e_x} + 4Nm\,\underline{e_y} + 2Nm\,\underline{e_z}$

$\underline{M}_h = -20Nm\,\underline{e_x} + 8Nm\,\underline{e_y} + 2Nm\,\underline{e_z}$

$a = 120\,mm,\ b = 800\,mm$



A(0.12 | 0 | 0) m
B(0 | 0 | 0) m
P(−0.8 | 0 | 0) m

Ortsvektoren: "Spitze minus Schaft"

$\underline{r}_{xy}$ ← Spitze, ← Schaft

$$\underline{r}_{AP} = \begin{pmatrix}0.12\\0\\0\end{pmatrix}m - \begin{pmatrix}-0.8\\0\\0\end{pmatrix}m = \begin{pmatrix}0.92\\0\\0\end{pmatrix}m$$

$$\underline{r}_{BP} = \begin{pmatrix}0\\0\\0\end{pmatrix}m - \begin{pmatrix}-0.8\\0\\0\end{pmatrix}m = \begin{pmatrix}0.8\\0\\0\end{pmatrix}m$$

$$\underline{F}_{res} = \begin{pmatrix}-50N\\80N\\-158N\end{pmatrix} + \begin{pmatrix}-20N\\60N\\-250N\end{pmatrix} = \underline{\begin{pmatrix}-70N\\140N\\-408N\end{pmatrix}}$$

$$\underline{M}_{res_P} = \underline{r}_{AP} \times \underline{F}_t + \underline{r}_{BP} \times \underline{F}_h + \underline{M}_t + \underline{M}_h$$

$$= \begin{vmatrix} e_x & e_y & e_z \\ +0.92 & 0 & 0 \\ -50 & 80 & -158 \end{vmatrix} + \begin{vmatrix} e_x & e_y & e_z \\ +0.8 & 0 & 0 \\ -20 & 60 & -250 \end{vmatrix} + \underline{M}_t + \underline{M}_h$$

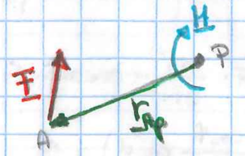
$$= \begin{pmatrix}0\,Nm\\+145.36\,Nm\\+73.6\,Nm\end{pmatrix} + \begin{pmatrix}0\\200\,Nm\\+48\,Nm\end{pmatrix} + \underline{M}_t + \underline{M}_h$$

$$= \begin{pmatrix}0\\+345.36\,Nm\\+121.6\,Nm\end{pmatrix} + \begin{pmatrix}-6\,Nm\\4\,Nm\\2\,Nm\end{pmatrix} + \begin{pmatrix}-20\,Nm\\8\,Nm\\3\,Nm\end{pmatrix} = \%$$

Moment einer Kraft: $\underline{M} = \underline{r} \times \underline{F}$ nicht kommutativ Reihenfolge!

Vektor $\underline{r}$ zeigt dabei vom Drehpunkt zum Kraftangriffspunkt.

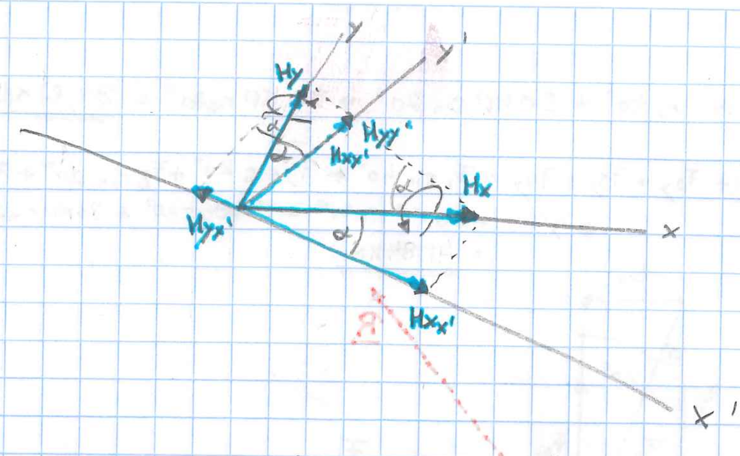
Also:



$$\% = \underline{\begin{pmatrix}-26\\357.36\\126.6\end{pmatrix} Nm}$$

Bsp. 3:

$M_x = 20\,kNm$
$M_y = 30\,kNm$
$\alpha = 25°$



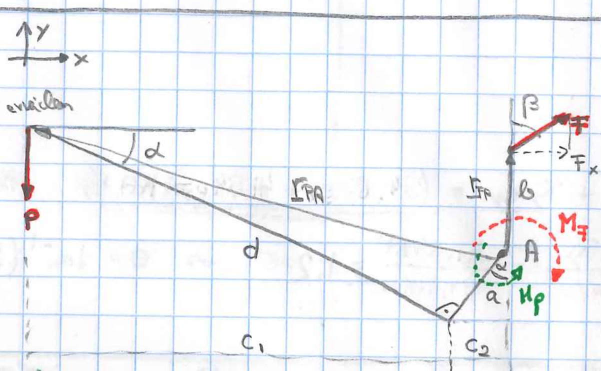
$$\left.\begin{aligned} M_{xx'} &= M_x \cdot \cos\alpha = 20\,kNm \cdot \cos(25°) = 18.126\,kNm \\ M_{yx'} &= M_y \cdot \sin\alpha = 30\,kNm \cdot \sin(25°) = 12.679\,kNm \end{aligned}\right\} \quad M_{resx'} = M_{xx'} - M_{yx'} = \underline{5.45\,kNm}$$

$$\left.\begin{aligned} M_{xy'} &= M_x \cdot \sin\alpha = 20\,kNm \cdot \sin(25°) = 8.45\,kNm \\ M_{yy'} &= M_y \cdot \cos\alpha = 30\,kNm \cdot \cos(25°) = 27.19\,kNm \end{aligned}\right\} \quad M_{resy'} = M_{xy'} + M_{yy'} = \underline{35.64\,kNm}$$

Bsp. 4:

Idee: $M_P \stackrel{!}{=} M_F$ um mittels $P \Rightarrow F$ zu erzielen

$F = 500\,N$
$a = 40\,mm$, $\alpha = 20°$
$b = 75\,mm$, $\beta = 60°$
$d = 350\,mm$



$M_F = F_x \cdot b = F \cdot \sin\beta \cdot b = 500N \cdot \sin(60°) \cdot 75mm = 32\,475.95\,Nmm$ auch $\underline{M_F} = \underline{r_{FA}} \times \underline{F}$

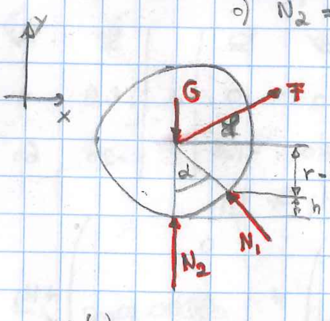
$c_1 = d \cdot \cos\alpha = 350mm \cdot \cos(20°) = 328.89\,mm$ $\underline{M_P} = \underline{r_{PA}} \times \underline{P}$

$c_2 = a \cdot \sin\alpha = 40mm \cdot \sin(20°) = 13.68mm$

$$M_P = P \cdot (c_1 + c_2) \stackrel{!}{=} M_F \longrightarrow P = \frac{M_F}{c_1 + c_2} = \frac{32\,475.95\,Nmm}{328.89mm + 13.68mm} = \underline{\underline{94.8\,N}}$$

Bsp. 5:

Vorgehensweise: o) Nur Normalkräfte als Kontaktkräfte da reibungsfrei.
o) $N_2 = 0$ beim Hochziehen, weil Kontakt verloren geht.



Berechne F für beliebiges φ & bestimme dann φ* für minimale Kraft.

Gleichgewichtsbed.: $\sum_i F_{x,i} = 0: F \cdot \cos\varphi - N_1 \cdot \sin\alpha = 0$

$\sum_i F_{y,i} = 0: F \cdot \sin\varphi + N_1 \cdot \cos\alpha - G = 0$

aus (x) $\rightarrow$ $N_1 = F \cdot \dfrac{\cos\varphi}{\sin\alpha}$ in (y): $F \cdot \sin\varphi + F \cdot \dfrac{\cos\varphi}{\sin\alpha} \cos\alpha - G = 0$

$$F\left(\sin\varphi + \frac{\cos\varphi}{\sin\alpha}\cos\alpha\right) = G$$

$$F = \frac{\sin\alpha}{\sin\varphi \sin\alpha + \cos\varphi\cos\alpha}\, G = \underline{\frac{\sin\alpha}{\cos(\alpha - \varphi)}\, G}$$

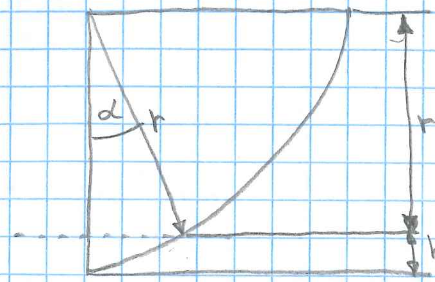
mit $\cos\alpha\cos\beta + \sin\alpha\sin\beta = \cos(\alpha-\beta) = \cos(\beta-\alpha)$!

F_{min} erreicht wenn Nenner maximal wird, d.h.

$$\cos(\alpha-\varphi^*) \overset{!}{=} 1 \rightarrow \underline{\varphi^* = \alpha} \quad \text{da } \cos(0)=1$$

$$\rightarrow \boxed{F_{min} = G\cdot\sin\alpha}$$

aus der Skizze:



ist abzulesen, dass $\cos\alpha = \frac{r-h}{r}$ also $\underline{\underline{\alpha = \arccos\frac{r-h}{r}}}$ gilt.

Bsp. 6:

Ortsvektor von B nach C:

$$\underline{r}_{CB} = \overset{C}{\begin{pmatrix}3\\1\\2\end{pmatrix}}m - \overset{B}{\begin{pmatrix}6\\4\\0\end{pmatrix}}m = \begin{pmatrix}-3\\-3\\2\end{pmatrix}m$$

Betrag: $|\underline{r}_{CB}| = \sqrt{(-3)^2+(-3)^2+2^2}\,m = \sqrt{22}\,m = \underline{4.69\,m}$

Einheitsvektor: $\underline{e}_{CB} = \frac{\underline{r}_{CB}}{|\underline{r}_{CB}|} = \begin{pmatrix}-3/4.69\\-3/4.69\\2/4.69\end{pmatrix} = \begin{pmatrix}-0.64\\-0.64\\0.43\end{pmatrix}$

Kraftvektor von F_1: $\underline{F}_1 = F_1\,\underline{e}_{CB} = 8N\cdot\begin{pmatrix}-0.64\\-0.64\\0.43\end{pmatrix} = \underline{\begin{pmatrix}-5.12\\-5.12\\3.41\end{pmatrix}N} \left(= -5.12\underline{e}_x - 5.12\underline{e}_y + 3.41\underline{e}_z\right)N$

Kraftvektor von F_2: $\underline{F}_2 = F_2\,\underline{e}_z = 10N\begin{pmatrix}0\\0\\1\end{pmatrix} = \underline{\begin{pmatrix}0\\0\\10\end{pmatrix}N} \left(= 10\,\underline{e}_z\,N\right)$

Resultierende Kraft: $\underline{\underline{R}} = \underline{F}_1 + \underline{F}_2 = \begin{pmatrix}-5.12\\-5.12\\3.41\end{pmatrix}N + \begin{pmatrix}0\\0\\10\end{pmatrix}N = \underline{\underline{\begin{pmatrix}-5.12\\-5.12\\13.41\end{pmatrix}N}}$

Resultierendes Moment: $\underline{M}^R_{(0)} = \underline{r}_{B0}\times\underline{F}_1 + \underline{r}_{A0}\times\underline{F}_2$

$$\underline{r}_{B0} = \overset{B}{\begin{pmatrix}6\\4\\0\end{pmatrix}} - \overset{0}{\begin{pmatrix}0\\0\\0\end{pmatrix}} = \begin{pmatrix}6\\4\\0\end{pmatrix}m$$

$$\underline{r}_{A0} = \begin{pmatrix}0\\6\\0\end{pmatrix} - \begin{pmatrix}0\\0\\0\end{pmatrix} = \begin{pmatrix}0\\6\\0\end{pmatrix}m$$

$$= \begin{pmatrix}6\\4\\0\end{pmatrix}m \times \begin{pmatrix}-5.12\\-5.12\\3.41\end{pmatrix}N + \begin{pmatrix}0\\6\\0\end{pmatrix}m \times \begin{pmatrix}0\\0\\10\end{pmatrix}N$$

$$= \begin{vmatrix}\underline{e}_x & \underline{e}_y & \underline{e}_z\\ 6 & 4 & 0\\ -5.12 & -5.12 & 3.41\end{vmatrix} Nm + \begin{vmatrix}\underline{e}_x & \underline{e}_y & \underline{e}_z\\ 0 & 6 & 0\\ 0 & 0 & 10\end{vmatrix} Nm$$

$$= \begin{pmatrix}13.64\\-20.64\\-10.24\end{pmatrix}Nm + \begin{pmatrix}60\\0\\0\end{pmatrix}Nm$$

$$\boxed{\underline{M}^R_{(0)} = \begin{pmatrix}73.64\\-20.64\\-10.24\end{pmatrix}Nm}$$

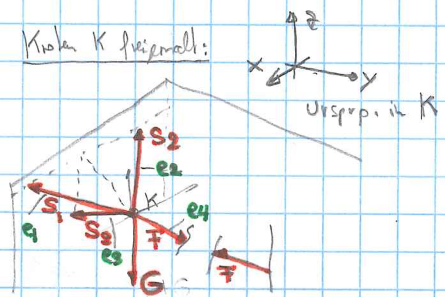
Betrag von $\underline{R}$: $|\underline{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(-5.12)^2 + (-5.12)^2 + 13.41^2}\ N$

$$\boxed{|\underline{R}| = 15.24\,N}$$

Bsp. 7: Vorgehensweise: Einheitsvektoren für die einzelnen Kräfte bestimmen & dann die Kräfte ins Glgw. setzen.

Glgw.: $\underline{S}_1 + \underline{S}_2 + \underline{S}_3 + \underline{F} + \underline{G} = \underline{0}$

Knoten K freigemacht:



wobei wieder die Einheitsvektoren benötigt werden:

$$\underline{e}_1 = \frac{1}{|\underline{r}_1|}\underline{r}_1 = \frac{1}{|\underline{r}_1|}\begin{pmatrix}2a\\-6a\\0\end{pmatrix} = \frac{1}{\sqrt{(2a)^2+(6a)^2}}\begin{pmatrix}2a\\-6a\\0\end{pmatrix}$$

$$\underline{e}_1 = \frac{1}{\underbrace{\sqrt{40}}_{\sqrt{4\cdot 10} = 2\sqrt{10}}\,a}\begin{pmatrix}2a\\-6a\\0\end{pmatrix} = \frac{1}{2\sqrt{10}\,a}\begin{pmatrix}2a\\-6a\\0\end{pmatrix} = \underline{\frac{1}{\sqrt{10}}\begin{pmatrix}1\\-3\\0\end{pmatrix}}$$

$$\underline{e}_2 = \frac{1}{|\underline{r}_2|}\begin{pmatrix}-2a\\-6a\\0\end{pmatrix} = \frac{1}{\underset{\text{s. oben}}{\sqrt{40}\,a}}\begin{pmatrix}-2a\\-6a\\0\end{pmatrix} = \frac{1}{2\sqrt{10}\,a}\begin{pmatrix}-2a\\-6a\\0\end{pmatrix} = \underline{\frac{1}{\sqrt{10}}\begin{pmatrix}-1\\-3\\0\end{pmatrix}}$$

$$\underline{e}_3 = \frac{1}{|\underline{r}_3|}\begin{pmatrix}0\\-6a\\-3a\end{pmatrix} = \frac{1}{\sqrt{(6a)^2+(3a)^2}}\begin{pmatrix}0\\-6a\\-3a\end{pmatrix} = \frac{1}{\underbrace{\sqrt{45}}_{\sqrt{5\cdot 9} = 3\sqrt{5}}\,a}\begin{pmatrix}0\\-6a\\-3a\end{pmatrix} = \frac{1}{3\sqrt{5}\,a}\begin{pmatrix}0\\-6a\\-3a\end{pmatrix} = \underline{\frac{1}{\sqrt{5}}\begin{pmatrix}0\\-2\\-1\end{pmatrix}}$$

$$\underline{e}_4 = \frac{1}{|\underline{r}_4|}\begin{pmatrix}-3a\\0\\-4a\end{pmatrix} = \frac{1}{\sqrt{(3a)^2+(4a)^2}}\begin{pmatrix}-3a\\0\\-4a\end{pmatrix} = \frac{1}{\underbrace{\sqrt{25}}_{5}\,a}\begin{pmatrix}-3a\\0\\-4a\end{pmatrix} = \underline{\frac{1}{5}\begin{pmatrix}-3\\0\\-4\end{pmatrix}}$$

D.h. die Richtungen der Kräfte sind jetzt bekannt, die Beträge aber noch nicht -> Beträge als Variablen einsetzen:

$$\underline{S}_1 = S_1\,\underline{e}_1 = \frac{S_1}{\sqrt{10}}\begin{pmatrix}1\\-3\\0\end{pmatrix}, \qquad \underline{F} = F\,\underline{e}_4 = \frac{F}{5}\begin{pmatrix}-3\\0\\-4\end{pmatrix} = \frac{G}{5}\begin{pmatrix}-3\\0\\-4\end{pmatrix}$$

$|\underline{F}|$ kann nur $|\underline{G}|$ sein, weil Seil (Schnitt)

$$\underline{S}_2 = S_2\,\underline{e}_2 = \frac{S_2}{\sqrt{10}}\begin{pmatrix}-1\\-3\\0\end{pmatrix}$$

$$\underline{S}_3 = S_3\,\underline{e}_3 = \frac{S_3}{\sqrt{5}}\begin{pmatrix}0\\-2\\-1\end{pmatrix}, \qquad \underline{G} = G\,(-\underline{e}_z) = G\begin{pmatrix}0\\0\\-1\end{pmatrix}$$

3 Unbekannte -> 3 Glg. nötig

Glgw. in $\underline{e}_x, \underline{e}_y, \underline{e}_z$:

$\underline{e}_x$: $\quad \frac{S_1}{\sqrt{10}} - \frac{S_2}{\sqrt{10}} - \frac{3}{5}G = 0 \qquad (1)$

$\underline{e}_y$: $\quad -\frac{3}{\sqrt{10}}S_1 - \frac{3}{\sqrt{10}}S_2 - \frac{2}{\sqrt{5}}S_3 = 0 \qquad (2)$

$\underline{e}_z$: $\quad -\frac{S_3}{\sqrt{5}} - \frac{4}{5}G - G = 0 \qquad (3)$

aus (3): $\frac{S_3}{\sqrt{5}} = -\left(\frac{4}{5} + \frac{5}{5}\right)G = -\frac{9}{5}G$

$$\boxed{S_3 = -\frac{9\sqrt{5}}{5}G} \qquad (4)$$

(4) in (2): $-\frac{3}{\sqrt{10}}S_1 - \frac{3}{\sqrt{10}}S_2 + \frac{2}{\sqrt{5}}\frac{9\sqrt{5}}{5}G = 0$

$$S_2 = -S_1 + \frac{\cancel{18}^{6}}{5}\frac{\sqrt{10}}{\cancel{3}}G = -S_1 + \frac{6\sqrt{10}}{5}G \qquad (5)$$

(5) in (1): $\frac{1}{\sqrt{10}}S_1 - \frac{1}{\sqrt{10}}\left(-S_1 + \frac{6\sqrt{10}}{5}G\right) - \frac{3}{5}G = 0$

$$\left(\frac{1}{\sqrt{10}} + \frac{1}{\sqrt{10}}\right)S_1 - \frac{1}{\cancel{\sqrt{10}}}\frac{6\cancel{\sqrt{10}}}{5}G - \frac{3}{5}G = 0$$

$$\frac{2}{\sqrt{10}}S_1 = \frac{9}{5}G$$

$$S_1 = \frac{9}{5}\frac{\sqrt{10}}{2}G \longrightarrow \boxed{S_1 = \frac{9\sqrt{10}}{10}G} \qquad (6)$$

(6) in (5) $S_2 = -\frac{9\sqrt{10}}{10}G + \frac{6\sqrt{10}}{5}G = \left(-\frac{9\sqrt{10}}{10} + \frac{12\sqrt{10}}{10}\right)G$

$$\boxed{S_2 = \frac{3\sqrt{10}}{10}G}$$