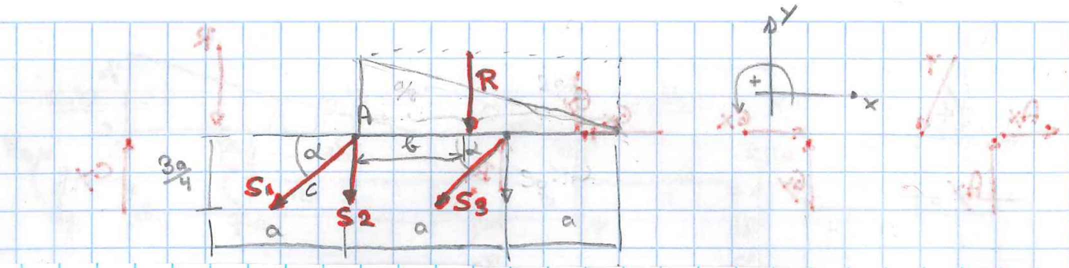
Bsp.: 17



Glgw.:

$$\sum_i M_{A_i} = 0: \quad -S_3 \cdot \sin\alpha \cdot a - R \cdot \frac{2}{3}a = 0 \qquad (1)$$

$$\sum_i F_{x_i} = 0: \quad -S_1 \cdot \cos\alpha - S_3 \cdot \cos\alpha = 0 \qquad (2)$$

$$\sum_i F_{y_i} = 0: \quad -S_1 \cdot \sin\alpha - S_2 - R - S_3 \cdot \sin\alpha = 0 \qquad (3)$$

$$c^2 = \left(\frac{3}{4}a\right)^2 + a^2$$

aus (1) $$-S_3 = R\frac{2}{3}\cancel{a}\frac{1}{\sin\alpha \cdot \cancel{a}} = \frac{2}{3}a \cdot q_0 \frac{1}{\sin\alpha} = \left| \sin\alpha = \frac{3a}{4c} = \frac{3a}{4}\left(\frac{9}{16}a^2 + a^2\right)^{-1/2} = \frac{3a}{4}\left(\frac{25}{16}a^2\right)^{-1/2} = \frac{3}{4}a\left(\frac{5}{4}a\right)^{-1} = \frac{3a}{4}\frac{4}{5a} = \frac{3}{5} \right|$$

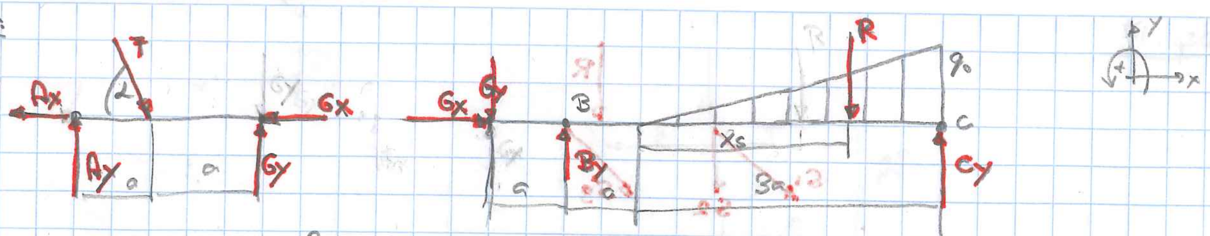
$$= \frac{2}{3}a q_0 \frac{5}{3} = \frac{10}{9}a q_0 \quad \Rightarrow \quad \boxed{S_3 = -\frac{10}{9}a q_0}$$

aus (2) $$\boxed{S_1 = -S_3 = \frac{10}{9}a q_0}$$

aus (3) $$S_2 = -S_1\frac{3}{5} - a \cdot q_0 - S_3\frac{3}{5} = \frac{3}{5}(-S_1 - S_3) - a \cdot q_0 = \left| S_1 = -S_3 \right| = -a \cdot q_0, \quad \boxed{S_2 = -a \cdot q_0}$$

$\rightarrow \frac{3}{5}(-S_1 + S_1) = 0$

Bsp. 18:



$$|R| = \int q(x)\,dx = \int_0^{3a} \frac{q_0 x}{3a}\,dx = \frac{q_0}{3a}\frac{x^2}{2}\Big|_0^{3a} = \frac{q_0}{6a} 9a^2 = \underline{\frac{3}{2} q_0 a}$$

$$,\; q(x) = \frac{q_0}{3a} x$$

(Geradengl. $y = kx + d$)

$$x_s = \frac{2}{3} 3a = \underline{2a}$$

Glgw. Gesamtsystem:

$$\sum_i Fx_i = 0: \quad -Ax + F\cdot\cos\alpha = 0 \quad \rightarrow \quad \underline{Ax = +F\cdot\cos\alpha}$$

Glgw. links:

$$\sum_i M_{A_i} = 0: \quad -F\cdot\sin\alpha\cdot a + Gy\cdot 2a = 0 \quad \rightarrow \quad \underline{Gy = +\frac{F\cdot\sin\alpha}{2}}$$

$$\sum_i Fx_i = 0: \quad -Ax + F\cdot\cos\alpha - Gx = 0 \quad \rightarrow \quad \underline{Gx} = Ax + F\cdot\cos\alpha = -F\cdot\cos\alpha + F\cdot\cos\alpha = \underline{0}$$

$$\sum_i Fy_i = 0: \quad Ay - F\cdot\sin\alpha + Gy = 0 \quad \rightarrow \quad \underline{Ay} = F\cdot\sin\alpha - \frac{F\cdot\sin\alpha}{2} = \underline{\frac{F\cdot\sin\alpha}{2}}$$

Glgw. rechts:

$$\sum_i M_{B_i} = 0: \quad +Gy\cdot a - R\cdot(a + x_s) + Cy\cdot 4a = 0 \qquad (1)$$

$$\sum_i M_{C_i} = 0: \quad +Gy\cdot 5a - By\cdot 4a + R\cdot(3a - x_s) = 0 \qquad (2)$$

aus (1):

$$Cy = \frac{1}{4a}\left(-Gy\,a + R\cdot 3a\right) = \frac{1}{4}\left(-\frac{1}{2}F\cdot\sin\alpha + \frac{3}{2}q_0 a\cdot 3\right)$$

$$\underline{Cy = \frac{1}{8}\left(9 q_0 a - F\cdot\sin\alpha\right)}$$

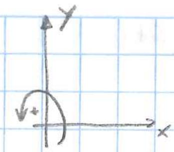
aus (2):

$$By = \frac{1}{4a}\left(Gy\,5a + R\,a\right) = \frac{1}{4}\left(5\frac{F\cdot\sin\alpha}{2} + \frac{3}{2}q_0 a\right)$$

$$\underline{By = \frac{1}{8}\left(5F\cdot\sin\alpha + 3 q_0 a\right)}$$

Bsp. 19:

links: mitte: rechts:



Mitte: $\sum_i M_{G1_i} = 0: \quad -F\cdot \cancel{a} + G_2 \cdot 2\cancel{a} = 0 \quad \rightarrow \quad \underline{G_2 = \frac{F}{2}}$

$\sum_i F_{y_i} = 0: \quad -G_1 + G_2 = 0 \quad \rightarrow \quad \underline{G_1 = G_2 = \frac{F}{2}}$

links: $\sum_i M_{A_i} = 0: \quad -M_A + G_1 \cdot a = 0 \quad \rightarrow \quad \underline{M_A = G_1 \cdot a = \frac{F}{2} a}$

$\sum_i F_{y_i} = 0: \quad A + G_1 = 0 \quad \rightarrow \quad \underline{A = -G_1 = -\frac{F}{2}}$

rechts: $\sum_i M_{B_i} = 0: \quad M_B + G_2 \cdot a + P \cdot a = 0 \rightarrow \underline{M_B = -(G_2 + P)a = -(F/2 + P)a}$

$\sum_i F_{y_i} = 0: \quad -G_2 - P + B = 0 \rightarrow \underline{B = G_2 + P = \frac{F}{2} + P}$

P unmittelbar links von G_2:

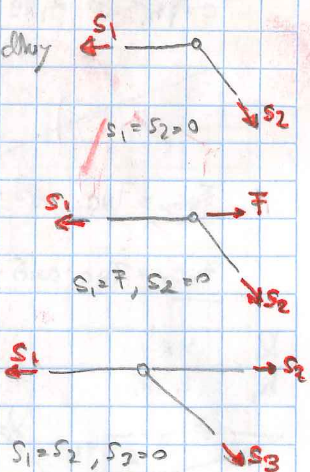
$\sum_i F_{y_i} = 0: \quad -G_1 - P + G_2 = 0 \rightarrow \underline{G_2 = G_1 + P = F/2 + P}$

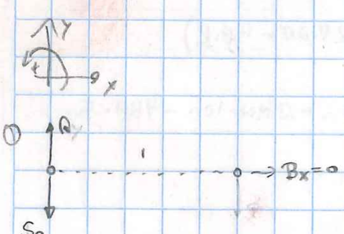
Bsp. 20: Nullstäbe:

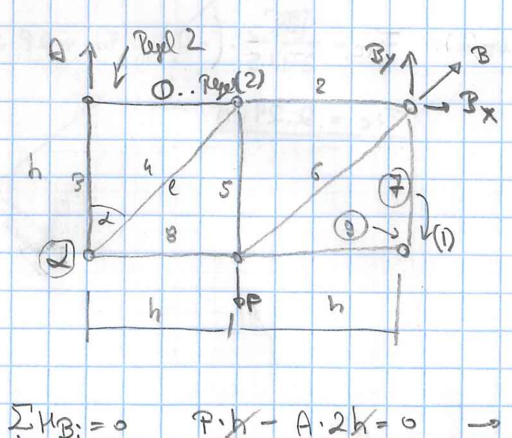
(1) Unbel. Knoten mit 2 Stäben die nicht in gl. Richtung zeigen → beide Nullstäbe.

(2) Bel. Knoten mit 2 Stäben + äußere Kraft in Richtung eines Stabes → anderer Nullstab.

(3) Unbel. Knoten mit 3 Stäben & 2 in gl. Richtg. → 3. ist Nullstab.







0... Nullstäbe

$l = \sqrt{h^2 + h^2} = \sqrt{2}\, h$

$S_{4y} = S_4 \cdot \cos\alpha = S_4 \cdot \frac{\cancel{h}}{\sqrt{2}\cancel{h}}$

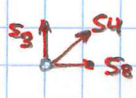
Gesamtsystem:

$\sum_i M_{B_i} = 0 \quad P \cdot \cancel{h} - A \cdot 2\cancel{h} = 0 \quad \rightarrow \quad \underline{A = \frac{P}{2}}$

$\sum_i M_{A_i} = 0 \quad B_y \cdot 2\cancel{h} - P \cdot \cancel{h} = 0 \quad \rightarrow \quad \underline{B_y = \frac{P}{2}}$

$B_x = 0$ aus Gesamtsystem.

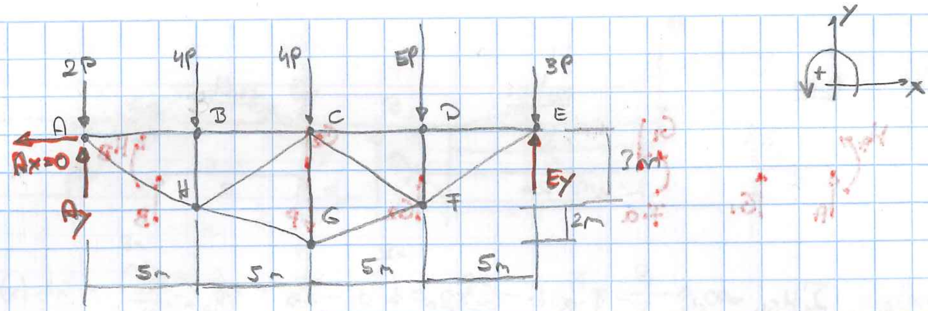
am Knoten ②



$S_3 = A \qquad \sum_i F_{y_i} = 0: \quad S_3 + S_{4y} = 0 \rightarrow S_{4y} = -S_3 = -A = -P/2$

$\rightarrow \frac{S_4}{\sqrt{2}} = -P/2 \rightarrow \underline{S_4 = -\frac{P\sqrt{2}}{2}}$

Bsp. 2b:



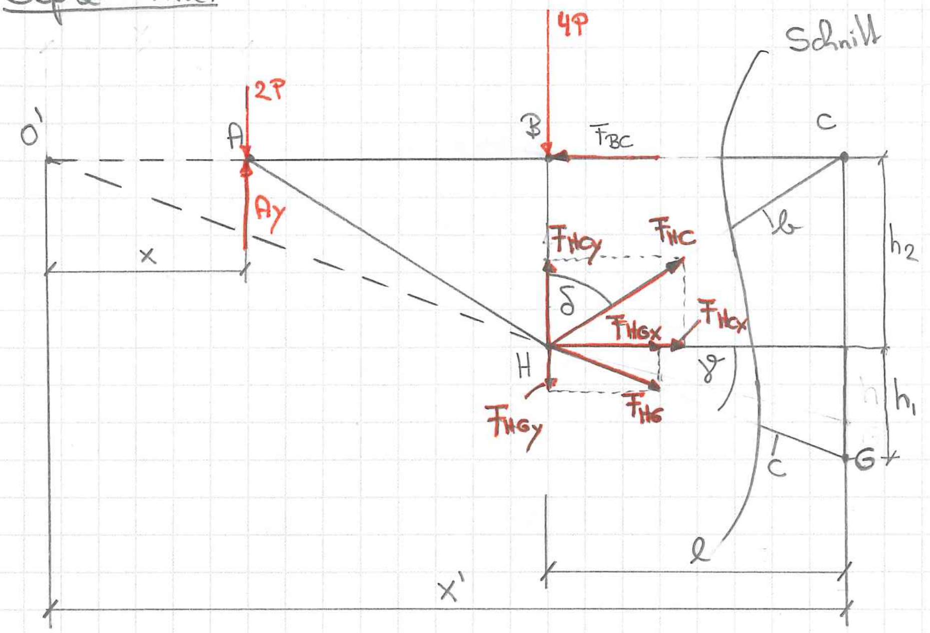
Gesamtsystem:

$$\sum_i M_{Ei} = 0: \quad -A_y \cdot 4\ell + 2P \cdot 4\ell + 4P \cdot 3\ell + 4P \cdot 2\ell + 5P \cdot \ell = 0$$

$$-4A_y \cdot \cancel{\ell} + 33P \cdot \cancel{\ell} = 0$$

$$\underline{\underline{A_y}} = \frac{33}{4} P = \underline{\underline{8.25\ \text{kN}}}$$

Bsp 21 Skizze:



Ablaufplan:

(1) Schnitt B–C, H–C, H–G

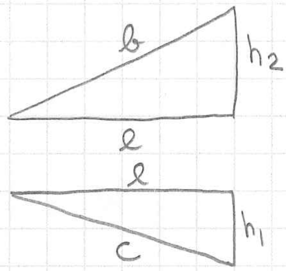
(2) Knotenkräfte einzeichnen

(3) Geometrie

(4) Komponenten der Knotenkräfte

Geometrie:

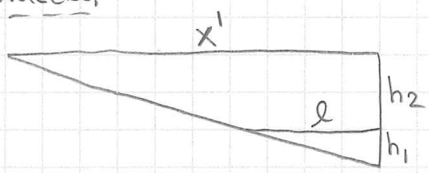
Satz von Pythagoras:



$$b = \sqrt{h_2^2 + l^2} = \sqrt{9+25} = \underline{\sqrt{34}\ m}$$

$$c = \sqrt{h_1^2 + l^2} = \underline{\sqrt{29}\ m}$$

ähnliche Dreiecke:



$$\frac{l}{h_1} = \frac{x'}{h_1 + h_2}$$

$$x' = \frac{l}{h_1}(h_1 + h_2) = \underline{12.5m}$$

damit gilt: $x = x' - 2l = \underline{2.5m}$

Kraftkomponenten:

$$F_{HGx} = F_{HG} \cdot \cos\gamma = F_{HG}\frac{l}{c} = \frac{5}{\sqrt{29}} F_{HG}$$

$$F_{HGy} = F_{HG} \cdot \sin\gamma = F_{HG}\frac{h_1}{c} = \frac{2}{\sqrt{29}} F_{HG}$$

$$F_{HCx} = F_{HC} \cdot \sin\delta = F_{HC}\frac{l}{b} = \frac{5}{\sqrt{34}} F_{HC}$$

$$F_{HCy} = F_{HC} \cdot \cos\delta = F_{HC}\frac{h_2}{b} = \frac{3}{\sqrt{34}} F_{HC}$$

[μnx]

Topic

Location | Time | Date

Momentenbilanzen:

$$\sum_i M_{H_i} = 0: \quad -A_y \cdot l + 2P \cdot l + F_{BC} \cdot h_2 = 0 \qquad (1)$$

$$\sum_i M_{C_i} = 0: \quad -A_y \cdot 2l + 2P \cdot 2l + 4P \cdot l + F_{HGx} \cdot h_2 + F_{HGy} \cdot l = 0 \qquad (2)$$

$$\sum_i M_{0'_i} = 0: \quad A_y \cdot x - 2P \cdot x - 4P \cdot (x+l) + F_{HCx} \cdot h_2 + F_{HCy} \cdot (x+l) = 0 \qquad (3)$$

aus (1): $F_{BC} = \frac{l}{h_2}\left(A_y - 2P\right) = \underline{\underline{10.42\,KN}}$ (Druckkraft auf B)

aus (2): $F_{HGx} \cdot h_2 + F_{HGy} \cdot l = 2A_y l - 8P \cdot l$

$$F_{HG} \underbrace{\left(\frac{l}{c} \cdot h_2 + \frac{h_1}{c} \cdot l\right)}_{(h_1 + h_2)\frac{l}{c}} = 2A_y l - 8P \cdot l$$

$$\underline{\underline{F_{HG}}} = \frac{c}{(h_1 + h_2) \cdot l}\left(2A_y \cdot l - 8P \cdot l\right) = \underline{\underline{9.15\,KN}}$$

aus (3): $F_{HCx} \cdot h_2 + F_{HCy} \cdot (x+l) = P(6x + 4l) - A_y x$

$$F_{HC} \underbrace{\left(\frac{l}{b} \cdot h_2 + \frac{h_2}{b}(x+l)\right)}_{\left(\frac{2l}{b} + \frac{x}{b}\right) h_2 \, = \, \frac{2l+x}{b} h_2}$$

$$\underline{\underline{F_{HC}}} = \frac{b}{(2l+x) h_2}\left[P(6x+4l) - A_y \cdot x\right] = \underline{\underline{2.24\,KN}}$$

analog für DC, CF, CG

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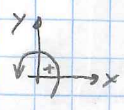
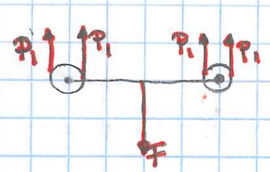
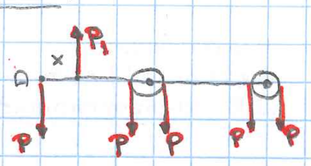
Bsp. 22:



linker Teil:



rechter Teil:



Glgw. links:

$\sum_i F_{y_i} = 0: \quad 4P_1 - F = 0$

$\rightarrow \underline{P_1} = \frac{F}{4} = \frac{800N}{4} = \underline{200N}$

Glgw. rechts:

$\sum_i F_{y_i} = 0: \quad -5P + P_1 = 0$

$\rightarrow \underline{P} = \frac{P_1}{5} = \frac{200N}{5} = \underline{40N}$

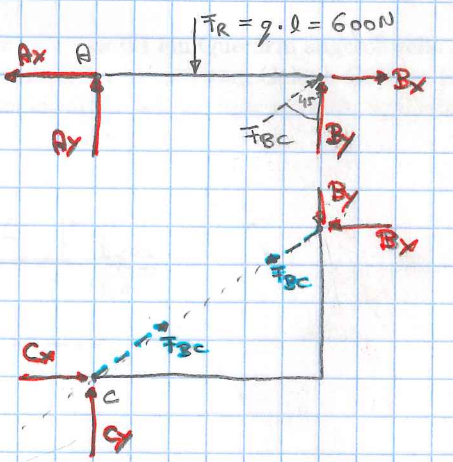
$\sum_i M_{A_i} = 0: \quad P_1 \cdot x - P \cdot (3r - r) - P \cdot (3r + r) - P \cdot (3r + 4r - r) - P \cdot (3r + 4r + r) = 0$

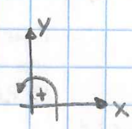
$P_1 \cdot x - P\left[2r + 4r + 6r + 8r\right] = 0$

$P_1 x - P\,20r = 0$

$\rightarrow \underline{x} = \frac{P\,20r}{P_1} = \frac{40N \cdot 1200mm}{200N} = \underline{240mm}$

Bsp. 23: BC ist Zweikraftstab!





oben

$\sum_i M_{A_i} = 0: \quad -F_R \cdot l/2 + B_y \cdot l = 0$

$\rightarrow \underline{B_y} = \frac{F_R}{2} = \underline{300N}$

$\sum_i F_{y_i} = 0: \quad A_y - F_R + B_y = 0$

$\rightarrow A_y = F_R - B_y = \underline{300N}$

$\sum_i F_{x_i} = 0: \quad -A_x + B_x = 0$

$A_x = B_x$

---

unten

$\sum_i M_{C_i} = 0: \quad B_x \cdot l - B_y \cdot l = 0$

$\rightarrow \underline{B_x} = B_y = \underline{300N} \quad \rightarrow \underline{A_x = 300N}$

$\sum_i F_{y_i} = 0: \quad \underline{C_y} = B_y = \underline{300N}$

$\sum_i F_{x_i} = 0: \quad \underline{C_x} = B_x = \underline{300N}$