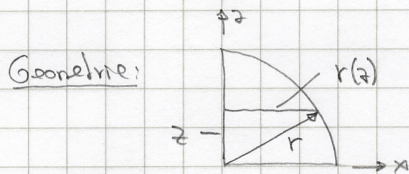


Aufgabe 13:

Allg.: $m = \int_m dm = \int_V \rho(z) dV$

$\rho(z) = Kz$ Integration über z

MMP-Werte: $\bar{z} = \frac{\int_m z dm}{\int_m dm}$



$$r^2 = z^2 + r^2(z) \rightarrow r^2(z) = r^2 - z^2$$

Gesamtmasse:

$$\begin{aligned} m &= \int_V \rho(z) dV = \int_z \rho(z) A(z) dz = \int_0^r Kz (r^2 - z^2) \pi dz = \int_0^r Kz r^2 \pi dz - \int_0^r Kz^3 \pi dz \\ &= Kr^2 \frac{z^2}{2} \pi \Big|_0^r - K \frac{z^4}{4} \pi \Big|_0^r = \frac{Kr^4 \pi}{2} - \frac{Kr^4 \pi}{4} = \underline{\underline{\frac{1}{4} Kr^4 \pi}} \end{aligned}$$

$$\begin{aligned} \text{MMP: } \int_m z dm &= \int_V z \rho(z) dV = \int_0^r Kz^2 (r^2 - z^2) \pi dz = \int_0^r Kr^2 z^2 \pi dz - \int_0^r Kz^4 \pi dz \\ &= \frac{Kr^2 z^3}{3} \pi \Big|_0^r - \frac{Kz^5}{5} \pi \Big|_0^r = Kr^5 \pi \left(\frac{1}{3} - \frac{1}{5} \right) = \underline{\underline{\frac{2}{15} Kr^5 \pi}} \end{aligned}$$

$$\bar{z} = \frac{\int_m z dm}{\int_m dm} = \frac{2Kr^5 \pi \frac{1}{15}}{Kr^4 \pi \frac{1}{4}} = \underline{\underline{\frac{8}{15} r}}$$