Aufgabe 14:

Allg.:
$$\bar{x} = \frac{\int_A x\,dA}{\int_A dA}, \quad \bar{y} = \frac{\int_A y\,dA}{\int_A dA}, \quad dA = y\,dx = \left(h - \frac{h}{a^n}x^n\right)dx$$

$$\int_A dA = \int_0^a y\,dx = \int_0^a h\,dx - \int_0^a \frac{h}{a^n}x^n\,dx = hx\Big|_0^a - h\frac{x^{n+1}}{a^n(n+1)}\Big|_0^a = h\left(a - \frac{a^{n+1}}{a^n(n+1)}\right)$$

$$= h\,\frac{a^{n+1}(n+1) - a^{n+1}}{a^n(n+1)} = h\,a^{n+1}\frac{n\cancel{+1-1}}{a^n(n+1)} = \underline{\underline{h\,a\,\frac{n}{n+1}}}$$

$$\int_A x\,dA = \int_0^a xh\,dx - \int_0^a \frac{h}{a^n}x^{n+1}\,dx = \frac{hx^2}{2}\Big|_0^a - \frac{h\,x^{n+2}}{a^n(n+2)}\Big|_0^a = h\left(\frac{a^2}{2} - \frac{x^{n+2}}{a^n(n+2)}\right)$$

$$= h\,\frac{a^{n+2}(n+2) - 2a^{n+2}}{2a^n(n+2)} = h\,a^{n+2}\frac{n\cancel{+2-2}}{2a^n(n+2)} = \underline{\underline{h\,a^2\,\frac{n}{2(n+2)}}}$$

$$\underline{\underline{\bar{x}}} = \frac{\cancel{h}a^{\cancel{2}}\cancel{n}(n+1)}{2(n+2)\cancel{ha}\cancel{n}} = \underline{\underline{\frac{n+1}{2(n+2)}\,a}}$$

$$\int_A y\,dA = \int_A y\,dy\,dx = \int_0^a \frac{y^2}{2}\,dx = \frac{1}{2}\int_0^a \left(h - \frac{h}{a^n}x^n\right)^2 dx = \frac{1}{2}\int_0^a \left(h^2 - 2\frac{h^2}{a^n}x^n + \frac{h^2}{a^{2n}}x^{2n}\right)dx$$

$$= \frac{h^2}{2}\left[\int_0^a dx - \frac{2}{a^n}\int_0^a x^n\,dx + \frac{1}{a^{2n}}\int_0^a x^{2n}\,dx\right] = \frac{h^2}{2}\left[a - \frac{2a^{n+1}}{a^n(n+1)} + \frac{a^{2n+1}}{a^{2n}(2n+1)}\right]$$

$$= \frac{h^2}{2}\left(\frac{a^{n+1}(n+1) - 2a^{n+1}}{a^n(n+1)} + \frac{a^{2n+1}}{a^{2n}(2n+1)}\right) = \frac{h^2}{2}\left(\frac{\cancel{a^n}\cdot a(n+1-2)}{\cancel{a^n}(n+1)} + \frac{\cancel{a^{2n}}\cdot a}{\cancel{a^{2n}}(2n+1)}\right)$$

$$= \frac{h^2}{2}\left(\frac{n-1}{n+1} + \frac{1}{2n+1}\right)a = \frac{h^2}{2}\,\frac{(2n+1)(n-1) + (n+1)}{(n+1)(2n+1)}\,a = \frac{h^2 a}{2}\,\frac{2n^2\cancel{-2n+n-1+n+1}}{(n+1)(2n+1)}$$

$$= h^2 a\,\frac{n}{(n+1)(2n+1)}$$

$$\underline{\underline{\bar{y}}} = \frac{h^{\cancel{2}}\cancel{a}\,n^{\cancel{2}}\cancel{(n+1)}}{\cancel{(n+1)}(2n+1)\cancel{ha}\cancel{n}} = \underline{\underline{\frac{n}{2n+1}\,h}}$$

oder $x = x(y)$ ausdrücken und über dy von 0 bis h integrieren.