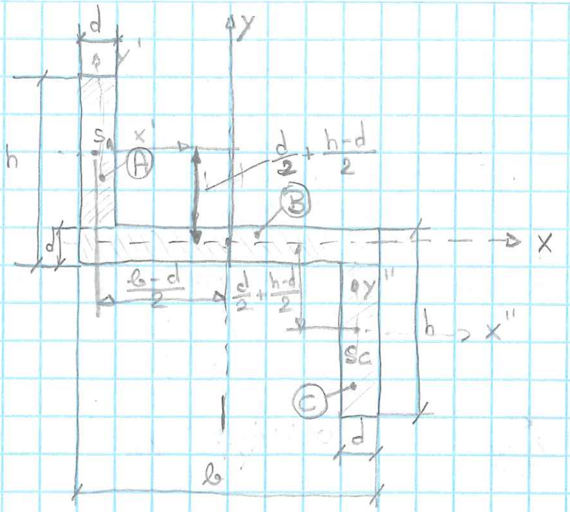
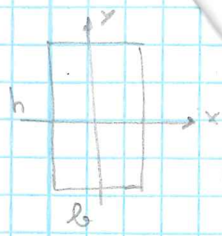
Bsp. 43:



Rechteck: $I_x = \frac{b h^3}{12}$ $I_y = \frac{b^3 h}{12}$



Rechteck A/C (Symmetrie):

$$I_x^{(A)} = I_{x'}^{(A)} + A d_y^2 = \frac{d(h-d)^3}{12} + d(h-d)\left(\frac{d}{2}+\frac{h-d}{2}\right)^2 = I_x^{(C)}$$

$$= 2.25\cdot 10^8_{mm^4} + 1.2\cdot 10^9 mm^4$$

$$\underline{I_x^{(A)} = I_x^{(C)} = 1.425\cdot 10^9 mm^4}$$

$$I_y^{(A)} = I_{y'}^{(A)} + A d_x^2 = \frac{d^3(h-d)}{12} + d(h-d)\left(-\frac{b-d}{2}\right)^2 = I_x^{(C)}$$

$$= 2.5\cdot 10^7_{mm^4} + 1.875\cdot 10^9 mm^4$$

$$\underline{I_y^{(A)} = I_y^{(C)} = 1.875\cdot 10^9 mm^4}$$

Trägheitsmomente

$$I_{xy}^{(A)} = \underbrace{I_{x'y'}^{(A)}}_{=0} + A d_x d_y = d(h-d)\left(\frac{d}{2}+\frac{h-d}{2}\right)\left(-\frac{b-d}{2}\right)$$

da Symmetrie

Deviationsmoment

$$\underline{I_{xy}^{(A)} = I_{xy}^{(C)} = -1.5\cdot 10^9 mm^4}$$

Rechteck B:

$$I_x^{(B)} = \frac{b d^3}{12} \qquad I_y^{(B)} = \frac{b^3 d}{12}$$

$$\underline{I_x^{(B)} = 5\cdot 10^7 mm^4} \qquad I_y^{(B)} = 1.8\cdot 10^9 mm^4$$

$\underline{I_{xy}^{(B)} = 0}$ aus Symmetriegründen

Gesamt:

$$I_x = I_x^{(A)} + I_x^{(B)} + I_x^{(C)} = 2 I_x^{(A)} + I_x^{(B)} \qquad \boxed{I_x = 2.9\cdot 10^9 mm^4}$$

$$I_y = I_y^{(A)} + I_y^{(B)} + I_y^{(C)} = 2 I_y^{(A)} + I_y^{(B)} \qquad \boxed{I_y = 5.6\cdot 10^9 mm^4}$$

$$I_{xy} = 2 I_{xy}^{(A)} \qquad \boxed{I_{xy} = -3\cdot 10^9 mm^4}$$

Hauptträgheitsmomente:

wenn $I_{xy} = \int xy\,dA$! sonst $(+)$

Drehwinkel: $\tan 2\Theta_p = \dfrac{-2 I_{xy}}{I_x - I_y}$ um in die Hauptlage zu gelangen.

Maximales- & Minimales Moment:
(Hauptträgheitsmoment)

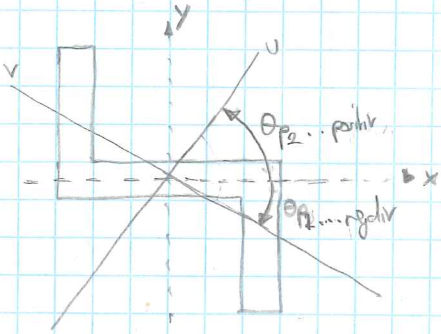
$$I_{max,min} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

oder charakteristische Polynom → Eigenwertglg., Hauptwerte sind EW des Tensors
$(\underline{A} - \lambda \underline{\underline{1}})\,\underline{n} = 0 \rightarrow \det(\underline{A} - \lambda \underline{\underline{1}}) = 0$

Bsp. 44:

$\tan 2\Theta_p = \dfrac{-2I_{xy}}{I_x - I_y} = -2.22 \rightarrow 2\Theta_{p_1} = -65.77°$ bzw. $2\Theta_{p2} = 2\Theta_{p_1} + 180° = 114.2°$

also: $\Theta_{p_1} = -32.9°$, $\Theta_{p2} = 57.1°$

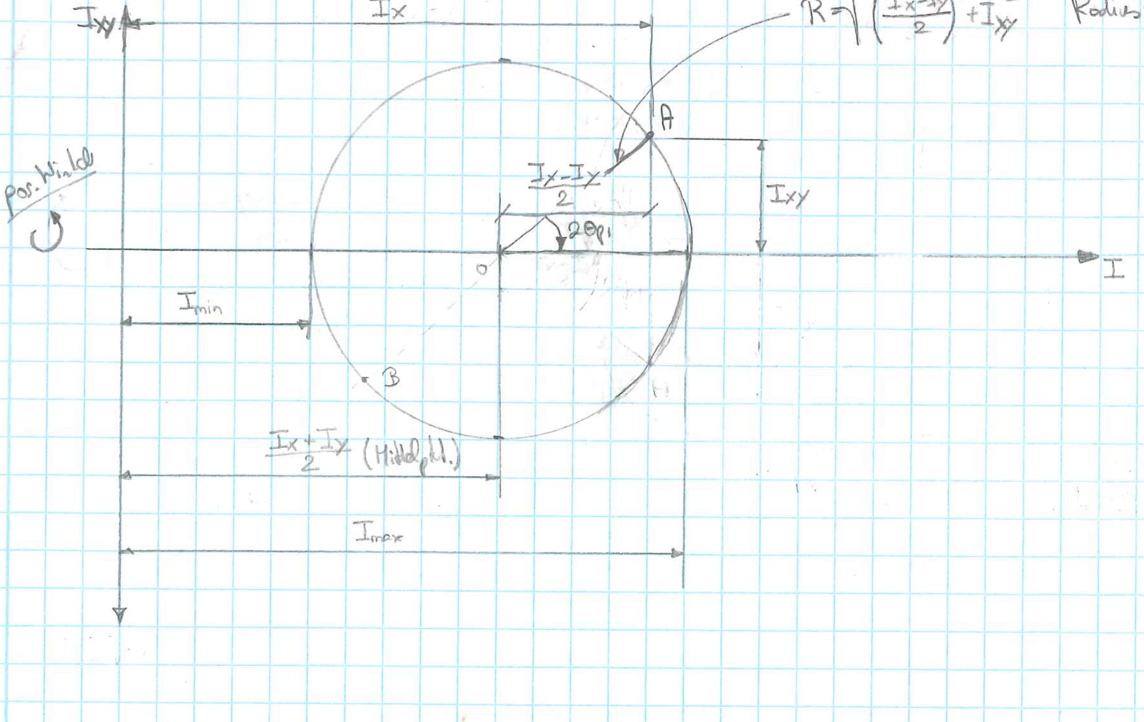


$$I_{max,min} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$= 4.25 \cdot 10^9\,mm^4 \pm 3.289 \cdot 10^9\,mm^4 \longrightarrow$ $\boxed{I_{max} = 7.54 \cdot 10^9\,mm^4 \quad I_{min} = 0.96 \cdot 10^9\,mm^4}$

Mohr'scher Trägheitskreis:

$R = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$ Radius



Bsp. 45:

Lösungen aus Bsp. 43:

$I_x = 2.9 \cdot 10^9\ mm^4$
$I_y = 5.6 \cdot 10^9\ mm^4$
$I_{xy} = +\,3 \cdot 10^9\ mm^4$

Konstruktion des Kreises:

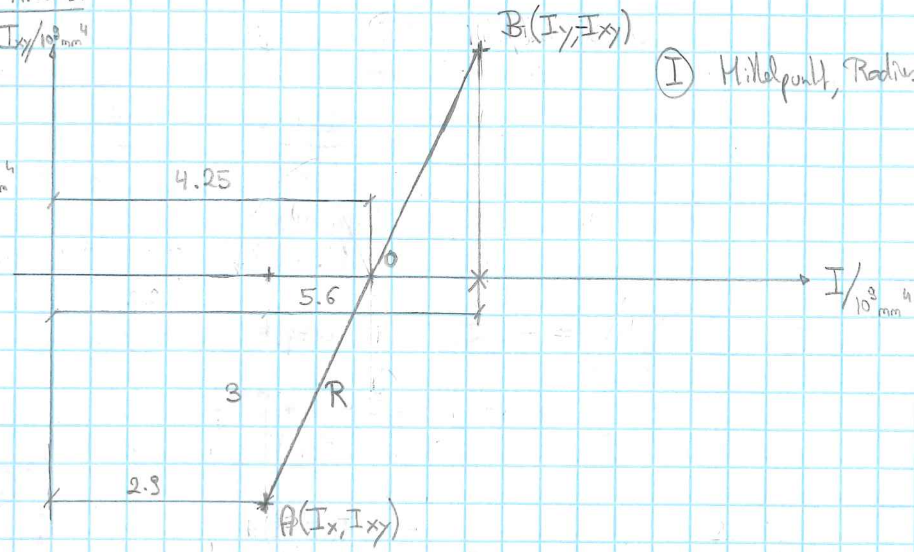
Mittelpunkt

$$M = \frac{I_x + I_y}{2} = 4.25 \cdot 10^9\ mm^4$$

$$R = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} = 3.29 \cdot 10^9\ mm^4$$

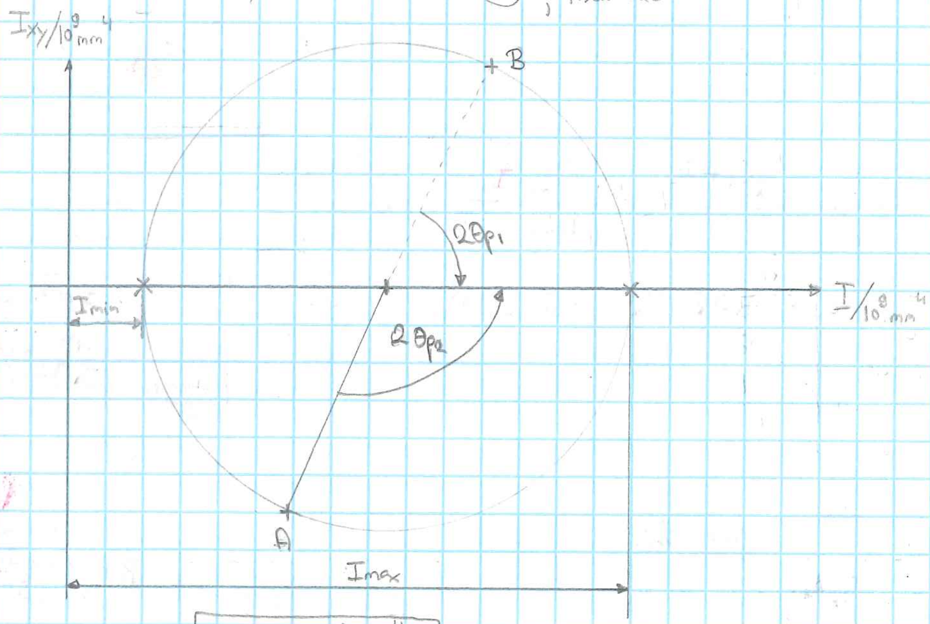
(Kontrolle)

(I) Mittelpunkt, Radius



pos. Winkel

(II), Kreis zeichnen



pos. Winkel

abgelesen: $\boxed{I_{min} \approx 1 \cdot 10^9\ mm^4}$
$\boxed{I_{max} \approx 7.5 \cdot 10^9\ mm^4}$

$2\theta_{p2} \approx 114°$
$\hookrightarrow \underline{\theta_{p2} \approx 57°}$ ✓ gegen UZS in Richtung positiver I-Achse

(vgl. Bsp. 44)

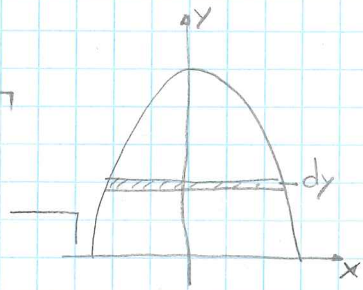
$2\theta_{p1} \approx -66°$
$\hookrightarrow \underline{\theta_{p1} \approx -33°}$ ✓

Bsp. 46:

$$I_x = \int_A y^2\, dA = \left|\, dA = x\,dy \,\right| = 2\int_0^h y^2 \sqrt{4-y}\; dy = (*)$$

re. & li. ← Symmetrie

$$y = h - h\frac{x^2}{b^2} \rightarrow x^2 = 4 - y \longrightarrow x = \sqrt{4-y}$$
$$= 4 - x^2$$

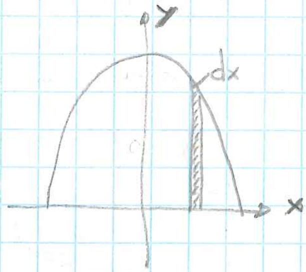


Mathematica / Tabelle:

$$\boxed{\int x^2\sqrt{4-x}\, dx = -\frac{2}{105}(4-x)^{3/2}\left[3x(5x+16)+128\right]}$$

$$(*) = -2\frac{2}{105}(4-y)^{3/2}\left[3y(5y+16)+128\right]\Big|_0^4 = 0 + 39.00 \Rightarrow \boxed{I_x = 39\,m^4}$$

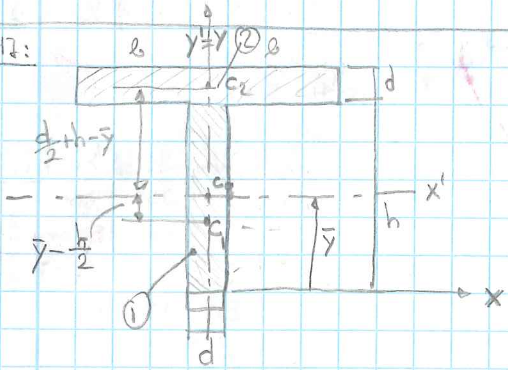
$$I_y = \int_A x^2\, dA = 2\int_0^b x^2 \underbrace{\overbrace{(4-x^2)\,dx}^{dA = y\,dx}}_{y} = 2\int_0^b 4x^2 dx - 2\int_0^b x^4 dx$$



$$= 2\left[4\frac{x^3}{3} - \frac{x^5}{5}\right]\Big|_0^2 = 2\left(\frac{32}{3} - \frac{32}{5}\right) = 8.53\,m^4$$

$$\Longrightarrow \boxed{I_y = 8.53\,m^4}$$

Bsp. 47:



Schwerpunkt:

$$\bar{y} = \frac{\sum \bar{y} A}{\sum A} = \frac{\overbrace{\frac{h}{2}\cdot d\cdot h}^{①} + \overbrace{\left(h+\frac{d}{2}\right)\cdot 2b\cdot d}^{②}}{\underbrace{d\cdot h}_{①} + \underbrace{2b\cdot d}_{②}}$$

$$\bar{y} = \frac{1.5625\cdot 10^6 + 4.125\cdot 10^6}{1.25\cdot 10^4 + 1.5\cdot 10^4} = \underline{206.8\,mm}$$

Flächenträgheitsmomente:

$$I_{x'} = \overbrace{\frac{d\cdot h^3}{12}}^{①} + \overbrace{d\cdot h\cdot\left(\bar{y}-\frac{h}{2}\right)^2}^{① \text{Steiner}} + \overbrace{\frac{2b\cdot d^3}{12}}^{②} + \overbrace{2b\cdot d\left(\frac{d}{2}+h-\bar{y}\right)^2}^{② \text{Steiner}}$$

$$= 6.51\cdot 10^7 + 8.364\cdot 10^7 + 3.125\cdot 10^6 + 6.98\cdot 10^7$$

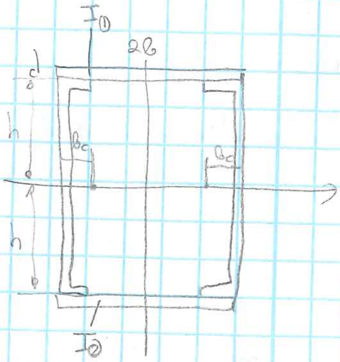
$$\underline{\underline{I_{x'} = 222\cdot 10^6\, mm^4}}$$

$$I_{y'} = \overbrace{\frac{d^3 h}{12}}^{①} + \overbrace{\frac{(2b)^3\cdot d}{12}}^{②}$$

$$= 2.604\cdot 10^6 + 1.125\cdot 10^8$$

$$\underline{\underline{I_{y'} = 115\cdot 10^6\, mm^4}}$$

Bsp. 48:



$$I_x = \underbrace{I_① + I_②}_{=2\frac{1}{12}2bd^3} + \underbrace{2d \cdot 2b \cdot \left(\frac{d}{2}+h\right)^2}_{\text{Steiner}} + 2 \cdot \bar{I}_{\bar{x}}$$

liegt auf Achse

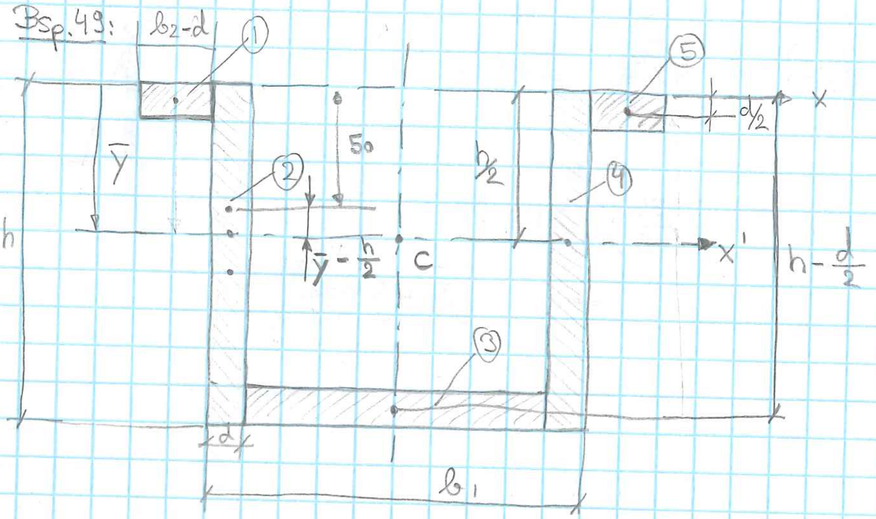
$$= 2\left[\frac{1}{12}2bd^3 + 2bd\left(\frac{d}{2}+h\right)^2 + \bar{I}_{\bar{x}}\right]$$

$$I_x = 2\left[1 + 1323 + 349\right] \rightarrow \boxed{I_x = 3.35 \cdot 10^3\,cm^4}$$

$$I_y = 2\left[\frac{1}{12}d(2b)^3\right] + 2\left[\bar{I}_{\bar{y}} + A_c(b-b_c)^2\right]$$

$$= 2\left[144 + 9.23 + 11.8 \cdot (6-1.28)^2\right] \rightarrow \boxed{I_y = 832\,cm^4}$$

Bsp. 49:



Schwerpunkt:

i	$\tilde{x}_i$	A_i	$\tilde{x}_i \cdot A_i$
1	$d/2$	$d(b_2-d)$	$d/2 \cdot d(b_2-d)$
2	$h/2$	$h \cdot d$	$h/2 \cdot h \cdot d$
3	$h-\frac{d}{2}$	$(b_1-2d) \cdot d$	$(h-d/2)(b_1-2d) \cdot d$
4	$h/2$	$h \cdot d$	$h/2 \cdot h \cdot d$
5	$d/2$	$d(b_2-d)$	$d/2 \cdot d(b_2-d)$

Zahlenwerte:

i	$\tilde{x}_i/mm$	A_i/mm^2	$\tilde{x}_i \cdot A_i/mm^3$
1	5	200	1000
2	50	1000	50 000
3	95	600	57 000
4	50	1000	50 000
5	5	200	1000
Σ	/	3000	159 000

$$\bar{y} = \frac{\sum_i \tilde{x}_i A_i}{\sum_i A_i} = \underline{\underline{53\,mm}}$$

Flächenträgheitsmomente:

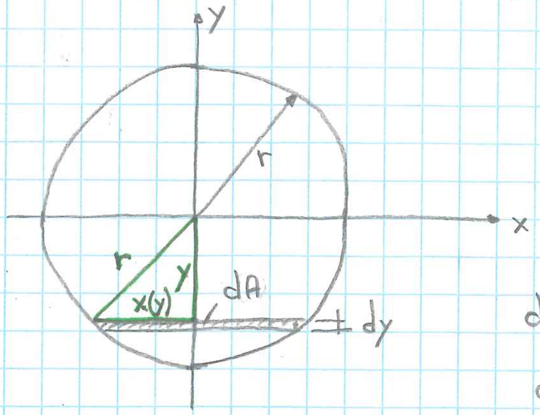
i	$\tilde{I}_{x_i}$	$A_i \cdot d_y^2$
1	$\frac{(b_2-d)d^3}{12}$	$d(b_2-d)(\bar{y}-\frac{d}{2})^2$
2	$\frac{d \cdot h^3}{12}$	$h \cdot d \cdot (\bar{y}-\frac{h}{2})^2$
3	$\frac{(b_1-2d) \cdot d^3}{12}$	$(h-\frac{d}{2})(b_1-2d)(h-\bar{y})^2$
4	$\frac{d \cdot h^3}{12}$	$h \cdot d \cdot (\bar{y}-\frac{h}{2})^2$
5	$\frac{(b_2-d)d^3}{12}$	$d(b_2-d)(\bar{y}-\frac{d}{2})^2$

Zahlenwerte:

i	$\tilde{I}_{x_i}/mm^4$	$A_i \cdot d_y^2/mm^4$	$\tilde{I}_{x_i} + A_i d_y^2/mm^4$
1	$1666.\dot{6}$	460 800	$462466.\dot{6}$
2	$833333.\dot{3}$	9000	$842333.\dot{3}$
3	5000	1325400	1330400
4	$833333.\dot{3}$	9000	$842333.\dot{3}$
5	$1666.\dot{6}$	460 800	$462466.\dot{6}$
Σ	/	/	$3.940 \cdot 10^6$

$$\underline{\underline{I_{x'} = 3.940 \cdot 10^6\,mm^4}}$$

Bsp. 50: Flächenträgheitsmoment Kreis



$$dA = 2\,x(y)\,dy \quad \text{mit} \quad r^2 = x(y)^2 + y^2, \quad x(y) = \sqrt{r^2 - y^2}$$

$$dA = 2\sqrt{r^2-y^2}\,dy$$

$$I_{xx} = \int_A y^2\,dA = \left|\, dA \text{ siehe oben} \right|$$

$$= \int_{-r}^{r} y^2\, 2\sqrt{r^2-y^2}\,dy = \left|\begin{array}{l}\text{Substitution der}\\ \text{Wurzel}\end{array}\right| = \left[-\frac{1}{2}y\sqrt{(r^2-y^2)^3} + \frac{r^2}{4}\left[y\sqrt{r^2-y^2} + r^2\arcsin\left(\frac{y}{r}\right)\right]\right]_{-r}^{r}$$

$$= \cancel{-\frac{1}{2}r\sqrt{\underbrace{(r^2-r^2)^3}_{=0}}} + \frac{r^2}{4}\left[\cancel{r\sqrt{\underbrace{r^2-r^2}_{=0}}} + r^2\arcsin\left(\frac{r}{r}\right)\right] + \cancel{\frac{1}{2}(-r)\sqrt{\underbrace{(r^2-(-r)^2)^3}_{=0}}}$$

$$- \frac{(-r)^2}{4}\left[(-r)\sqrt{\underbrace{r^2-(-r)^2}_{=0}} + (-r)^2\arcsin\left(\frac{-r}{r}\right)\right]$$

$$= \frac{r^4}{4}\arcsin(1) - \frac{r^4}{4}\arcsin(-1) \qquad \text{wobei} \qquad \begin{array}{l}\arcsin(1) = \frac{\pi}{2}\\ \arcsin(-1) = -\frac{\pi}{2}\end{array}$$

$$= \frac{r^4\pi}{8} + \frac{r^4\pi}{8} = \frac{2r^4\pi}{8}$$

$$\left\| I_{xx} = \frac{r^4\pi}{4} \right\| \quad , \quad I_{yy} = I_{xx} \ \dots \text{ Symmetrie}$$

Polar: $I_p = I_{xx} + I_{yy} = \dfrac{r^4\pi}{2} = \dfrac{d^4\pi}{32}$

⚠ Weg über Polar nach Axial!